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Abstract

Fantastic beasts are magical creatures that cannot be seen unless one looks for them with the eye of the wizard, but that still play a significant role in the world. The fantastic beasts we hunt and find in the present paper are welfare changes induced by resource shocks that are invisible in quantitative trade models with monopolistic competition and heterogeneous firms if one relies on the pervasive assumption of demand exhibiting constant elasticity of substitution. We argue that, for fantastic beasts to materialize, markups have to vary across firms and firm heterogeneity has to vary across sectors. This is shown both theoretically and empirically exploiting a panel of 76 countries and 17 manufacturing industries for the period 1995-2020.

JEL Classification: F12, F43

Keywords: Quantitative trade models

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Fantastic Beasts and Where to Find Them*

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Abstract

Fantastic beasts are magical creatures that cannot be seen unless one looks for them with the eye of the wizard, but that still play a significant role in the world. The fantastic beasts we hunt and find in the present paper are welfare changes induced by resource shocks that are invisible in quantitative trade models with monopolistic competition and heterogeneous firms if one relies on the pervasive assumption of demand exhibiting constant elasticity of substitution. We argue that, for fantastic beasts to materialize, markups have to vary across firms and firm heterogeneity has to vary across sectors. This is shown both theoretically and empirically exploiting a panel of 76 countries and 17 manufacturing industries for the period 1995-2020.

KEY WORDS: Quantitative trade models, Variable Markups, Incomplete Pass-through, Resource Shocks, Immiserizing Growth.

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"[E]conomists should not emancipate themselves from the tyranny of Cobb-Douglas only to enchain themselves in a new Solow CES tyranny."

(Samuelson, 1965; p.346)

"[T]he amusing creatures described hereafter are fictional and cannot hurt you."

(Dumbledore, 2001; p.viii).

1 Introduction

Fantastic beasts are a magical creature that cannot be seen unless one looks for them with the eye of the wizard, but still play a significant role in the world, sometimes a fascinating one, other times a dangerous one. The fantastic beasts we are after in the present paper are welfare changes induced by shocks that one fails to see in quantitative trade models with monopolistic competition and heterogeneous firms based on the pervasive assumption of demand exhibiting constant elasticity of substitution (Costinot and Rodriguez-Clare, 2014).

As Samuelson's metaphor implies, constant elasticity of substitution (CES) entails several intertwined restrictions. To see this, let us follow Nocco et al. (2024) and introduce some definitions. Call 'absolute markup' the difference between a firms' profit-maximizing price and its marginal cost, and 'relative markup' the ratio of the profit-maximizing price to the marginal cost. Then use 'absolute pass-through' to refer to the derivative of the profit-maximizing price to the marginal cost, and 'relative pass-through' to refer to the corresponding percentage change, that is, the derivative of the logarithm of the profit-maximizing price to the logarithm of the marginal cost. Under CES, the relative markup, the absolute pass-through and the relative pass-through are all constant and common across firms. Only the absolute markup varies and increases with marginal cost, which implies that in equilibrium it is larger for less productive firms as these have higher marginal cost. In addition, both the absolute and the relative pass-throughs are also constant and common across firms. However, while the former is larger than one, the latter is equal to one (which is what the literature refers to as 'complete pass-through'). We show that a minimal departure from CES allowing for variable relative markup, while retaining constant though incomplete absolute pass-through (IPT), is enough to unveil unexpected welfare effects.

The model we rely on features an arbitrary number of sectors that differ in terms of firm heterogeneity and an arbitrary number of countries that differ in terms of their comparative advantage across sectors. It is Ricardian in the sense that labor is the only input and comparative advantage is driven by differences in unit labor requirements. In particular, average requirements vary across sectors, but may still exhibit some statistical overlaps across them due to random dispersion around the mean as in Melitz (2003).

The shocks we focus on are 'resource shocks', which we introduce following a classical interpretation in the spirit of Houthakker, 1955, Rybczynski, 1955, Solow, 1956, 1957 and Jones, 1971, according to which, despite that in a Ricardian setup labor is the only explicit input, one may think that there are implicitly other "missing factors ", which consist of country- and sector-specific resources that are complementary to labor. These resources are available in fixed endowments, cannot be consumed, produced, nor used to finance entry, and cannot be traded. Complementarity implies that an exogenous increase in the endowment of a country's sector-specific resource determines a sector-biased outward shift of its production possibility frontier, which we will also refer to as "growth".

While our model is ready for full-fledged quantitative exercises based on calibration, validation and simulation of all kinds of counterfactual scenarios, in this paper we use its elegant properties in terms of sufficient statistics to construct resource shocks to specific sectors of a country's economy that have no impact on its welfare under CES but sizeable welfare effects under IPT. This can be done as long as sectors differ with regard to the concentration of their firms' unit labor requirements around the sectoral means, which we call "technological concentration"for short.

We target two types of fantastic beasts, which we refer to as "immiserizing growth"and "enriching decline". In the former case, a domestic resource increase that does not change welfare under CES leads to lower welfare under IPT. In the latter case, a domestic resource reduction that does not change welfare under CES leads to higher welfare under IPT. The reason for these divergences is that IPT allows for richer reallocation patterns between firms and sectors than CES does due to variable markups and incomplete pass-through. The constancy of the pass-through is not essential. However, together with the assumption that firms' labor input requirements are (inverse) Pareto distributed, it generates a simple expression of national welfare as a function of a very limited number of sufficient statistics. The Pareto assumption also buys the opportunity of measuring technological concentration through a single exogenous parameter.

Using "CES-neutral "to refer to a resource shock that does not change welfare under

CES, the main results can be summarized as follows. If an expansionary CES-neutral domestic resource shock hits a sector with low technological concentration, a country may still experience immiserizing growth, that is, welfare losses under IPT. Vice versa, if the contractionary CES-neutral domestic resource shock hits a sector with low technological concentration, the country may still experience enriching decline, that is, welfare gains under IPT. These results are derived both theoretically and empirically for resource shocks of realistic magnitude as proof of concept.

Our results contribute to two main lines of research. Firstly, they contribute to the literature on the gains from trade in quantitative trade models and, in particular, to the ongoing debate about "new gains from trade"in models with imperfect competition and firm heterogeneity (see, e.g., Arkolakis et al., 2012; Melitz and Redding, 2015; Arkolakis et al., 2019). The closest paper to ours is indeed Arkolakis et al., 2019. However, while our demand system belongs to the class of demand systems they use to quantify the pro-competitive effects of trade, to find our fantastic beasts one has to allow for technological concentration to differ across sectors, which they end up preventing by assumption. Moreover, our model brings income effects into quasi-linear models with constant absolute pass-through that have been used for trade policy analysis (see, e.g., Melitz and Ottaviano, 2008; Nocco et al., 2019; Nocco et al., 2024). Lastly, our model offers a quantifiable implementation of the setup with additive separable utility, income effects, variable markups and constant absolute pass-through recently put forth by Melitz et al., 2024 for analyzing non-discriminatory industrial policy. Secondly, our findings contribute to the vast literature on the effects of resources shocks, which have been investigated in various setups since early contributions on immiserizing growth (Bhagwati, 1958, Johnson, 1967, Bhagwati, 1968) and the Dutch disease (Corden and Neary, 1982), and more generally on the resource curse (see, e.g., Ploeg, 2011, and Ploeg and Poelhekke, 2019, for surveys). Our definition of "immiserizing growth"differs from the traditional one in that it refers to the welfare impact of a CES-neutral resource shock rather than a generic resource shock. In this respect, it is a relative rather than an absolute definition.

The rest of the paper is organized in five sections. Section 2 introduces the model. Without making the (inverse) Pareto assumption on the distribution of firms' unit labor requirements, it highlights some general properties of the chosen setup. Section 3 adds the Pareto assumption and derives the equilibrium of model, proving existence and uniqueness. It also characterizes the model's "welfare formula"and gravity equation, and its relation to the existing quantitative trade models. Section 4 analyzes the welfare effects of resource shocks and the necessary conditions for the fantastic beasts to materialize. Section 5 hunts and finds the fantastic beasts in a panel of 76 countries and 17 manufacturing industries in the period 1995-2020. Section 6 concludes.

2 A multi-country and multi-sector open economy

There are countably many countries and sectors: indexes $j = 1, ..., J$ indicate a country as a source of supply, indexes $l = 1, ..., J$ indicate a country as a source of demand, and indexes $z = 1, ..., Z$ indicate a sector. Consumption goods are traded across countries. In each country a continuum of varieties of a differentiated consumption good, indexed by $i \in [0, N_l(z)]$, is consumed; where $N_l(z)$ is the measure of varieties of goods in sector z available for consumption in country l.

Monopolistically competitive firms employ labor in one country and produce one variety in one sector with constant returns to scale. Labor is the only input, it is homogeneous, perfectly mobile across sectors but not mobile across countries. Firm entry is unrestricted but costly: producers willing to enter in a country j and sector z pay an exogenous sunk cost in terms of $f_j(z) > 0$ labor units, to develop a new technology in that country and sector pair. After this payment, a firm realizes its idiosyncratic conversion rate of labor per unit of output, as a random draw $c > 0$ from a continuous c.d.f. $G_j(c; z)$ that is specific to the country j and sector z.[1] After making a successful entry, firms producing in a country j might export to any other country l facing a sector-specific iceberg trade costs $\tau_{jl}(z) \geq 1$.

Call $N_j^E(z)$ the measure of entrants in country j sector z, then $M_j(z) \leq N_j^E(z)$ is the measure of varieties produced in country j of goods in sector z, and only a subset $N_{jl}(z) \leq M_j(z)$ of them is shipped to country l. Thus, the measure of available varieties (domestic and imported) in a certain market l is given by $N_l(z) \equiv \sum_{j=1}^{J} N_{jl}(z)$.

[1]The technological coefficient c is the inverse of labor productivity, i.e. it describes the ratio of units of labor per unit of output of a certain variety. Typically, the literature refers to this coefficient as marginal and average "cost" but in the present framework the wage will be determined in equilibrium, thus, the marginal and average cost at which a firm operates is endogenous, through the wage.

2.1 Consumers' behavior

In every country $l = 1, ..., J$, preferences are represented by a Cobb-Douglas aggregator across sectors $z = 1, ..., Z$, while consumption bundles of varieties within a sector are ranked by quadratic preferences:[2]

$$U_l = \prod_{z=1}^{Z} \left[\sum_{j=1}^{J} \left(\int_0^{N_{jl}(z)} \alpha q_{jl}^c (i; z) - \frac{\gamma}{2} q_{jl}^c (i; z)^2 \, di \right) \right]^{\beta(z)} \quad \alpha, \gamma > 0, \tag{1}$$

where $q_{jl}^c(i; z)$ is the quantity of good i from sector z produced in country j and consumed in country l and $\beta(z) \in (0, 1)$ such that $\sum_{z=1}^{Z} \beta(z) = 1$ are sector-specific shares. The sub-utility representing preferences across varieties within a sector is a special case of the class introduced by Bulow-Pfleiderer (1984), which in our notation would take the functional form

$$\alpha q_{jl}^c (i; z) - \frac{\gamma}{1 - \sigma} q_{jl}^c (i; z)^{1-\sigma} \ , \ \alpha \geq 0, \ \gamma \neq 0, \ \sigma < 1.$$

This class implies constant absolute pass-through $1/(1 - \sigma)$ from marginal cost to profit-maximizing price. It includes the quadratic case for $\alpha > 0$, $\gamma > 0$ and $\sigma = -1$, as well as the CES case for $\alpha = 0$, $\gamma < 0$ and demand elasticity to price equal to $1/\sigma > 1$. While we could develop our analysis for the general case, we prefer to focus on the quadratic one as it allows us to remain parsimonious in terms of parameters while still allowing for both constant absolute pass-through and variable demand elasticity to price.

Individual consumers in country l earning a wage $w_l > 0$ take the set of available varieties and prices as given and maximize (1) subject to the budget constraint

$$\sum_{z=1}^{Z} \sum_{j=1}^{J} \int_0^{N_{jl}(z)} p_{jl} (i; z) \, q_{jl}^c (i; z) \, di = w_l \ , \tag{2}$$

where $p_{jl} (i; z)$ is the price (at destination) of a variety i of goods from sector z produced

[2]Two remarks shall be noted. First, we discuss the case of a Cobb-Douglas aggregator across sectors for the sake of exposition, but the analysis goes through for every homothetic aggregator. Second, α is the marginal utility of any variety when its consumption is null. For $\alpha > 0$, the reader can simply think at $\alpha \equiv 1$ without loss of generality. In fact, as taste for consumption are not heterogeneous across varieties or countries, one parameter among α or γ is sufficient to represent the taste for differentiation relative to absolute willingness to pay, i.e. the ratio γ/α, in consumer's preferences. With a richer notation, variety, sector and country-and-sector-specific parameters $\alpha_l(i; z) > 0$ and $\gamma_l(i; z) > 0$ could be used to indicate within-sector patterns of vertical differentiation and horizontal differentiation, respectively. We abstract from these sources of discrimination, by willingness to pay (*quality*) and by country of origin (*Armington*).

in country j and sold to country l.

Between sectors, the marginal utility is unbounded. Therefore, every consumer of every country l will demand varieties from every sector z. Within sector, the marginal utility from consumption of a certain variety is finite ($\alpha > 0$ is necessary for this result). This implies that there is a choke price at which the optimal consumption of a variety is null. Let $\hat{p}_l(z) > 0$ be the price that implies zero demand in country l for a variety in sector z. The Marshallian individual demand function in country l for a variety of sector z sold in country l at a price p, regardless where production occurs, is given by:

$$q^*_{jl}(p; z) = \frac{\alpha}{\gamma}\left(1 - \frac{p}{\hat{p}_l(z)}\right), \quad \forall j. \tag{3}$$

Firm-level elasticity of demand to price is fully described by the relative price with respect to the choke price $\varepsilon_l(p; z) = \frac{p/\hat{p}_l(z)}{1-p/\hat{p}_l(z)}$ in absolute value, and we restrict the analysis to the case $\varepsilon_l(p; z) > 1$ for price-competition to arise. The ratio $\varepsilon_l(p; z)/(\varepsilon_l(p; z) - 1)$ defines the markup rate charged by a firm of sector z selling at a price p in country l, and it is given by:

$$mkp^*_{jl}(p; z) = \frac{1}{2 - \hat{p}_l(z)/p}, \quad \forall j, \tag{4}$$

hence, markup in a certain destination market vary with the relative price with respect to the choke price in the market. Markups are a decreasing function of the relative price $p/\hat{p}_l(z)$, thus, within a sector those varieties for which consumers exhibit greater demand are the ones sold at greater markup.

2.2 Firms' behavior

A firm located in country j, competing in sector z, endowed with a technological coefficient c, hires labor in the same country at a competitive wage w_j which is employed in a linear production function:

$$q_j(c; z) = \frac{\ell_j(c; z)}{c} \tag{5}$$

where $\ell_j(c; z)$ is the employment of labor at the firm. The marginal cost of production is $w_j c$. For goods that are produced in country j and shipped to a certain country l becomes the marginal cost of production and delivery is $\tau_{jl}(z) w_j c$.

Since consumers in a given country have the same income, the aggregate demand function in a certain destination l amounts to the individual demand (3) times the market size L_l. Thus, the marginal revenue in a destination l for a good of sector z at a price $p_{jl}(c;z)$ is given by $2p_{jl}(c;z) - \hat{p}_l(z)$. The equivalence of marginal revenue and marginal cost yields the price that maximizes profit for a firm of sector z producing in country j and selling to country l:

$$p_{jl}(c;z) = \frac{\hat{p}_l(z) + \tau_{jl}(z) w_j c}{2} . \tag{6}$$

Substituting in the Marshallian demand (3) shows that the technological coefficient that implies a zero demand in country l for a good of sector z produced in country j is

$$c^*_{jl}(z) = \frac{\hat{p}_l(z)}{\tau_{jl}(z) w_j} , \tag{7}$$

therefore, the marginal cost for profitably producing in country j and shipping to country l is bounded above by the choke price in the destination country l.

The measure of firms in country j producing in sector z that serve market l consists of the fraction of entrants in that country and sector, $N^E_j(z)$, whose technological coefficient does not exceed the export cutoff (7)

$$N_{jl}(z) = G_j(c^*_{jl}(z);z) N^E_j(z) . \tag{8}$$

Price $p_{jl}(c;z)$, markup factor $mkp_{jl}(p;z)$, output $q_{jl}(c;z)$, employment in production $\ell_{jl}(c;z)$, revenue $r_{jl}(c;z)$ and profit $\pi_{jl}(c;z)$ associated to the shipment from country j to country l of a firm with technological coefficient c in sector z are:

$$\begin{aligned}
p_{jl}(c;z) &= \frac{\hat{p}_l(z)}{2}\left(1 + \frac{c}{c^*_{jl}(z)}\right) \\
mkp_{jl}(c;z) &= \frac{1}{2}\left(1 + \frac{c^*_{jl}(z)}{c}\right) \\
q_{jl}(c;z) &= \frac{L_l}{2\gamma}\frac{\alpha}{c^*_{jl}(z)}\left(c^*_{jl}(z) - c\right) \\
\ell_{jl}(c;z) &= \frac{L_l}{2\gamma}\frac{\alpha\tau_{jl}(z)}{c^*_{jl}(z)}\left(c^*_{jl}(z)\,c - c^2\right) \\
r_{jl}(c;z) &= \frac{w_j L_l}{4\gamma}\frac{\alpha\tau_{jl}(z)}{c^*_{jl}(z)}\left(c^*_{jl}(z)^2 - c^2\right) \\
\pi_{jl}(c;z) &= \frac{w_j L_l}{4\gamma}\frac{\alpha\tau_{jl}(z)}{c^*_{jl}(z)}\left(c^*_{jl}(z) - c\right)^2 ,
\end{aligned} \tag{9}$$

where constant returns to scale in production and market segmentation allow all variables to be assigned by origin and destination pair.

Firm-level variables (9) are well-defined on $c \leq c^*_{jl}(z)$, i.e. such that a firm of sector z producing in country j with technological coefficient c charges an optimal price-at-destination in market l that does not exceed the choke price $\hat{p}_l(z)$. Firms under the same circumstances but endowed with $c > c^*_{jl}(z)$ optimally choose not to serve market l, hence, production, employment, revenue and profits are null and there is no contribution from these firms to the set of varieties available in country l.

2.3 Sectoral expenditure share and price concentration

A characteristic feature of this model is that consumer's expenditure and indirect utility are characterized by moments of the distribution of prices relative to the choke price. To see this, define the first and second moment of the distribution of prices relative to the choke price in country l sector z among goods shipped from country j:

$$\bar{p}_{jl}(z) \equiv N_{jl}(z)^{-1} \int_0^{N_{jl}(z)} \frac{p(i)}{\hat{p}_l(z)} di \quad \text{and} \quad \bar{\bar{p}}_{jl}(z) \equiv N_{jl}(z)^{-1} \int_0^{N_{jl}(z)} \left(\frac{p(i)}{\hat{p}_l(z)}\right)^2 di \, .$$

The optimal consumer's expenditure in country l on goods of sector z sourced from country j and the maximum sub-utility enjoyed by consuming in country l goods from sector z sourced from country j are, respectively:

$$\begin{aligned} e_{jl}(z) &\equiv \int_0^{N_{jl}(z)} p(i) q^*_{jl}(p(i); z) \, di = \hat{p}_l(z) \frac{\alpha}{\gamma} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z) \, , \\ u_{jl}(z) &\equiv \int_0^{N_{jl}(z)} \left(\alpha q^*_{jl}(p(i); z) - \frac{\gamma}{2} q^*_{jl}(p(i); z)^2\right) di = \frac{\alpha^2}{2\gamma} \left(1 - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z) \, . \end{aligned}$$

In the spirit of Dixit and Stiglitz (1977), it is insightful to recognize that expenditure of a consumer in country l on a bundle of goods from a certain sector $\sum_{j=1}^J e_{jl}(z)$ sourced anywhere can be decomposed into a quantity index $\mathbb{Q}_l(z) \equiv (1/\alpha) \sum_{j=1}^J u_{jl}(z)$ and a price index $\mathbb{P}_l(z) \equiv \sum_{j=1}^J e_{jl}(z)/\mathbb{Q}_l(z)$. The system of first order conditions of the consumer problem evaluated at the sectoral choke prices and the budget constraint across sectors determines the expenditure in goods of sector z in country l, that is $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta_l(z) w_l$, where the sector-specific expenditure share is given by:

$$\theta_l(z) \equiv \frac{\beta(z)\eta_l(z)}{\sum_{s=1}^Z \beta(s)\eta_l(s)} \in (0, 1) \, , \tag{10}$$

thus, it deviates from the exogenous Cobb-Douglas expenditure shares by means of the endogenous coefficient

$$\eta_l(z) \equiv \frac{\mathbb{P}_l(z)}{\hat{p}_l(z)} = \frac{2\sum_{j=1}^{J}\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)}{\sum_{j=1}^{J}\left(1 - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)} \in (0,1)\,, \tag{11}$$

that is an index of sector and country specific concentration of prices relative to the choke price. Therefore, non-homothetic preferences among varieties within sectors imply that sectoral expenditure shares are endogenous: they depend on the equilibrium level of concentration in the within-sector price distributions. In a given country l, the exogenous Cobb-Douglas shares, e.g., $\beta(z)$, correspond to the actual expenditure shares $\theta_l(z)$ if and only if sectors are characterized by the same concentration in price distributions, such that $\eta_l(z) = \eta_l$ for all z.

2.4 Equilibrium

An open economy equilibrium with diversification is characterized by a strictly positive measure of entrants in every country and sector pair, such that $N_j^E(z) > 0$ for all $j = 1, ..., J$ and $z = 1, ..., Z$. This implies that in every destination market there is a measure of domestic incumbent firms in each sector, such that $c_{ll}^*(z) > 0$ for all $l = 1, ..., J$ and $z = 1, ..., Z$, hence, the domestic cutoff $c_{ll}^*(z) \equiv c_l^*(z)$ determines the choke price $\hat{p}_l(z) = w_l c_l^*(z)$.

While an equilibrium with diversification is a common conjecture under CES preferences (i.e. with each variety granted a positive marginal utility by construction), a framework with finite choke prices implies that for some country and sector pairs there might not be a strictly positive measure of entrants. For this reason we will first define an open economy equilibrium with diversification in the context of this model and then discuss existence, uniqueness and properties of the equilibrium in the next section.

In every country and sector pair (j, z) with a strictly positive measure of entrants, free entry implies that the expected value of a new entry unconditional on being successful matches the entry cost:

$$\text{FEC}: \quad \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} \pi_{jl}(c;z) dG_j(c;z) = w_j f_j \qquad \forall (j,z). \tag{12}$$

such that there are no rents from firm ownership. Output market clearing requires that sales made by all firms in a sector z that serve a destination country l add up to the expenditure of that country in that sector:

$$\text{OMC}: \quad \sum_{j=1}^{J} N_{jl}(z) \int_0^{c_{jl}^*(z)} r_{jl}(c;z) \frac{dG_j(c;z)}{G_j(c_{jl}^*(z);z)} = \theta_l(z) w_l L_l \qquad \forall (l,z). \tag{13}$$

Labor market clearing requires that sales made by all firms producing in a country j are equal to aggregate labor income (from production and entry) made by workers in country j:

$$\text{LMC}: \quad \sum_{z=1}^{Z} \sum_{l=1}^{J} N_{jl}(z) \int_0^{c_{jl}^*(z)} r_{jl}(c;z) \frac{dG_j(c;z)}{G_j(c_{jl}^*(z);z)} = w_j L_j \quad \forall j. \tag{14}$$

Given a set of preference parameters $\{\alpha, \gamma, \{\beta(z)\}_{z=1}^{Z}\}$, market sizes $\{L_j\}_{j=1}^{J}$, entry costs $\{f_j\}_{j=1}^{J}$, a distribution of technological coefficients $\{G_j(c;z)\}_{j=1,z=1}^{J,Z}$ and a set of bilateral sector specific trade costs $\{\tau_{jl}(z)\}_{j=1,l=1,z=1}^{J,J,Z}$, the equilibrium of the model within the cone of diversification consists of:

a) a vector of wages $w_l > 0$ for every country $l = 1, 2, ...J$

b) a vector of choke prices $\hat{p}_l(z) = c_l^*(z) w_l > 0$ for every country $l = 1, 2, ...J$ and sector $z = 1, 2, ..., Z$

c) a vector of measures of entrants $N_j^E(z) > 0$ for every origin country $j = 1, 2, ...J$ and sector $z = 1, 2, ..., Z$

that satisfy

i) the system of $J \times Z$ free entry conditions (12),

ii) the system of $J \times Z$ sectoral output market clearing conditions (13),

iii) the system of J aggregate labor market clearing conditions (14),

once the export cutoff (7), the measure of exporters (8), the definitions of firm-level profit and revenue in (9), sectoral expenditure share (10) and sectoral price concentration (11) are understood. Without loss of generality, labor in one of the countries is taken as numeraire, such that the corresponding wage is 1 before and after any change in the fundamentals of the economy.

2.5 Welfare

Given prices $\boldsymbol{p}_l$ of available varieties in destination market l and a wage w_l, the maximum utility from consumption enjoyed by the representative consumer in country l is the Cobb-Douglas aggregator of the sectoral utility-based quantity indexes $V(\boldsymbol{p}_l, w_l) = \prod_{z=1}^{Z} \mathbb{Q}_l(z)^{\beta(z)}$. The budget constraint $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta_l(z)w_l$ implies

$$\mathbb{Q}_l(z) = \frac{\theta_l(z)}{\mathbb{P}_l(z)} w_l = \frac{\theta_l(z)}{\eta_l(z)} \frac{w_l}{\hat{p}_l(z)} = \frac{\beta(z)}{\bar{\eta}_l} \frac{w_l}{\hat{p}_l(z)},$$

where the coefficient $\bar{\eta}_l \equiv \sum_{z=1}^{Z} \beta(z)\eta_l(z) \in (0,1)$ is the weighted average of the sectoral price concentration indexes in country l, with weights given by the sectoral shares in consumer's preferences. Measuring welfare as maximized utility yields:

$$V(\boldsymbol{p}_l, w_l) = \bar{\eta}_l^{-1} \prod_{z=1}^{Z} \left(\frac{\beta(z)}{\hat{p}_l(z)} \right)^{\beta(z)} w_l = \bar{\eta}_l^{-1} \prod_{z=1}^{Z} \left(\frac{\beta(z)}{c_l^*(z)} \right)^{\beta(z)}, \tag{15}$$

where the last equality follows from the definition of choke price $\hat{p}_l(z) = c_l^*(z)w_l$ in an open economy equilibrium with diversification. Therefore, (15) shows two results: first, welfare in country l is a geometric average across sectors of the country's sectoral productivity cutoffs $1/c_l^*(z)$; second, given the same vector of choke prices and the same wage, through $\bar{\eta}_l$ welfare is higher when, on average, prices are more dispersed away from the sectoral choke price.

2.6 Gravity equation

The open economy equilibrium with diversification predicts a structural gravity representation of trade flows. To obtain this, start from the definition of expenditure, call $X_{jl}(z) = e_{jl}(z)L_l$ the value of imports of country l from country j in sector z and let $X_l(z) \equiv \sum_{j=1}^{J} X_{jl}(z)$ be the aggregate expenditure of country l in goods of sector z sourced from anywhere. Substituting for the measure of exporters $N_{jl}(z) = G_j(c_{jl}^*(z); z)N_j^E(z)$ as in (8) yields the gravity equation in terms of the measure of entrants in each country and sector:

$$X_{jl}(z) = \frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) G_j(c_{jl}^*(z); z)N_j^E(z)}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) G_m(c_{ml}^*(z); z)N_m^E(z)} X_l(z) \, .$$

Free entry (12) implies that the cost of entry in a country j sector z, that is $N_j^E(z)w_j f_j$, equals total profit in that sector and country. Let $\delta_j(z) = \Pi_j(z)/R_j(z) \in (0,1)$ be the fraction of aggregate profit $\Pi_j(z) \equiv N_j^E(z) \sum_{l=1}^J \int_0^{c_{jl}^*(z)} \pi_{jl}(c;z) dG_j(c;z)$ over aggregate revenue $R_j(z) \equiv N_j^E(z) \sum_{l=1}^J \int_0^{c_{jl}^*(z)} r_{jl}(c;z) dG_j(c;z)$ in country j sector z, such that $N_j^E(z)w_j f_j = \delta_j(z) R_j(z)$. Output market clearing (13) implies $X_l(z) = \theta_l(z) Y_l$, where $Y_l \equiv w_l L_l$ denotes income in country l.

Under free entry, total revenue coincides with total labor income (associated with both production and entry), thus, with total value added. Define $\rho_j(z) \in (0,1)$ the share of employment in sector z of country j. Then, labor market clearing (14) at the sectoral level yields $R_j(z) = \rho_j(z) w_j L_j$, that allows to substitute for $N_j^E(z) = \delta_j(z)\rho_j(z) L_j / f_j$. This completes the characterization of the gravity equation:

$$X_{jl}(z) = \left(\frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) G_j(c_{jl}^*(z);z)\delta_j(z)\rho_j(z)L_j/f_j}{\sum_{m=1}^J \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) G_m(c_{ml}^*(z);z)\delta_m(z)\rho_m(z)L_m/f_m} \right) \theta_l(z) Y_l \,, \qquad (16)$$

where the expression in brackets is the fraction of expenditure in goods of sector z that country l sources from country j.

With respect to the family of structural gravity equations that are popular in the quantitative trade literature one difference emerges: that is, the role of price dispersion by origin relative to the choke price at destination. Ceteris paribus, a country l sources relatively less from an origin j if this is characterized by a greater concentration of suppliers at the choke price $\bar{p}_{jl}(z) \approx \bar{\bar{p}}_{jl}(z) \approx 1$ than with a more dispersed set of prices $\bar{\bar{p}}_{jl}(z) < \bar{p}_{jl}(z) < 1$. A finite choke price is necessary to account for this channel. It is, however, not sufficient. In particular, in the next section we will show that assuming an Inverse Pareto distribution of technological coefficients makes such channel immaterial.

3 Implications of a Pareto distribution of technologies

Assume now that the distribution of unit labor requirements is an Inverse Pareto on the support $[0, c_j^{max}(z)]$, with a country- and sector-specific location parameter $c_j^{max}(z) > 0$ and a sector-specific shape parameter $k(z) > 1$ such that $G_j(c;z) = (c/c_j^{max}(z))^{k(z)}$. The mean of the distribution is $\frac{k(z)}{k(z)+1} c_j^{max}(z)$. For $k(z) \to 1$ the distribution becomes uniform (maximum dispersion), whereas for $k(z) \to \infty$ the distribution degenerates to a unit mass point at $c_j^{max}(z)$, describing maximum concentration at the upper bound of the support. Henceforth, we will refer to the parameter $k(z)$ as the technological concentration of sector z.

The Inverse Pareto assumption imposes a discipline on the distribution of prices relative to the choke price within a country-sector. The choke price in sector z country l is such that the firm-level demand is null, which corresponds to $c = c^*_{jl}(z)$ for a firm producing in any country j. The relative price is given by $\frac{p_{jl}(c;z)}{\bar{p}_l(z)} = \frac{1}{2}(1 + c/c^*_{jl}(z))$ and it is distributed over the support $c \in [0, c^*_{jl}(z)]$ according to the truncated Inverse Pareto $G^*_{jl}(c; z) = (c/c^*_{jl}(z))^{k(z)}$. As a consequence, the first and second moment of the relative price distribution

$$\bar{p}_{jl}(z) = \frac{2k(z)+1}{2(k(z)+1)} \equiv \mu_1(z) \tag{17}$$

$$\bar{\bar{p}}_{jl}(z) = \frac{2k(z)^2+4k(z)+1}{2(k(z)+2)(k(z)+1)} \equiv \mu_2(z) \tag{18}$$

are no longer endogenous, since they only depend on the parameter of technological concentration $k(z)$, thus, not on country characteristics of origin or destination. Consequences for welfare, equilibrium allocation and trade flows are far from being innocuous.

3.1 Implications for welfare

With first and second moment of the within-sector relative price distribution that are the same across countries, the concentration of prices (11) in equilibrium does not depend on the composition of varieties by country of destination. The coefficient describing price concentration

$$\eta(z) = \frac{2(\mu_1(z) - \mu_2(z))}{1 - \mu_2(z)} = \frac{2k(z)+2}{2k(z)+3}$$

is increasing in the exogenous technological concentration and it does not vary by country, i.e. $\eta_l(z) \equiv \eta(z)$. Thus, sectoral expenditure shares (10) are the same across countries, i.e. $\theta_l(z) \equiv \theta(z)$, and in those sectors with relatively more (less) concentrated distribution of technologies equilibrium expenditure shares are larger (smaller) than their corresponding Cobb-Douglas shares, i.e., $\beta(z)$.

This implies that welfare (15) is not affected by any endogenous response of price dispersion to the equilibrium adjustment of the economy. Hence, welfare analysis can be conducted without loss of generality only looking at the geometric average of sectoral productivity cutoffs:

$$W_l = \prod_{z=1}^{Z} \left(\frac{\beta(z)}{c_l^*(z)} \right)^{\beta(z)} \quad . \tag{19}$$

A comparison of the two welfare measures (15) and (19) shows that the adoption of an Inverse Pareto distribution of technology hides the role that endogenous responses in price dispersion might play for welfare. This conclusion relates to the work by Melitz and Redding (2015), who point out how in trade models with endogenous firm selection moments of the micro structure matter for welfare.

In our setup, before assuming an Inverse Pareto distribution of technologies, welfare grows proportionally with the average price dispersion, that is, $1/\bar{\eta}_l$. Therefore, the model shows how a shock that ceteris paribus leads to a lower price concentration (11) through endogenous selection of heterogeneous firms (8) magnifies the welfare gains. In contrast, after assuming an Inverse Pareto, that channel is lost, but in exchange of tractability that allows us to derive a rich set of analytical results.

3.2 Implications for equilibrium allocations and trade flows

Evaluating the free entry condition (12) given the Inverse Pareto distribution of technological coefficients yields:

$$\text{FEC*}: \quad \sum_{l=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} = 1 \ \ \forall (j,z) \ , \tag{20}$$

$$\text{where } c_j^{aut}(z) \equiv \left(\frac{c_j^{max}(z)^{k(z)}}{\zeta_\Pi(z) L_j / f_j} \right)^{\frac{1}{1+k(z)}}$$

is the cutoff cost in sector z of country j in autarky, that is for $\tau_{jl}(z) \to \infty$ for every $j \neq l$, and the coefficient $\zeta_\Pi(z) > 0$ is a decreasing transformation of technological concentration $k(z)$; see appendix A for a detailed derivation. Output market clearing (13) evaluated with an Inverse Pareto distribution yields:

$$\text{OMC*}: \quad \sum_{j=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} w_j f_j N_j^E(z) = w_l f_l N_l^{E\ aut}(z) \quad \forall (l,z) \tag{21}$$

$$\text{where } N_l^{E\ aut}(z) = \theta(z)\delta(z)\frac{L_l}{f_l}$$

is the measure of entrants in sector z of country l in autarky and $\delta(z) \in (0,1)$ is the profit-to-revenue ratio, that is decreasing in $k(z)$ and - given the Inverse Pareto distribution of technologies - is constant across countries; see appendix A for a detailed derivation. Labor market clearing (14), given the Inverse Pareto distribution of technological coefficients becomes:

$$\text{LMC*}: \quad \sum_{z=1}^{Z} \left(\frac{f_j N_j^E(z)}{\delta(z)} \sum_{l=1}^{J} \frac{L_l / L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} \right) \quad = \quad L_j \quad \forall j \tag{22}$$

where the left hand side shall be interpreted as the aggregate labor demand in country j. Moments of the relative price distribution ($\bar{p}_{jl}(z)$ and $\bar{\bar{p}}_{jl}(z)$), expenditure shares ($\theta_l(z)$), profit-to-revenue ratios ($\delta_j(z)$) do not depend on country specific characteristics, hence they do not affect distance in within-sector trade flows. The structural gravity equation simplifies to:

$$X_{jl}(z) = \left(\frac{\left(\tau_{jl}(z) w_j c_j^{max}(z)\right)^{-k(z)} \rho_j(z) L_j / f_j}{\sum_{m=1}^{J} \left(\tau_{ml}(z) w_m c_m^{max}(z)\right)^{-k(z)} \rho_m(z) L_m / f_m} \right) \theta(z) Y_l \,. \tag{23}$$

Therefore, the model predicts a canonical gravity equation of trade flows, with an import elasticity equal to $-k(z)$, while allowing for variable markups and sectoral choke prices.[3]

3.3 Uniqueness and existence of the equilibrium

As cost cutoffs and measure of entrants in the case of prohibitively high trade costs are determined in closed form, we rely on them to characterize the equilibrium allocations in open economy relative to their analogues in autarky. This can be achieved by defining two sets of changes in variables and three sets of bundling parameters.

In the case of variables, define $x_l(z)$ as the trade-induced change in cost cutoff for sector z in country l, and $y_j(z)$ the trade-induced change in the measure of entrants in

[3] A gravity equation has been typically associated with both a Pareto distribution of productivity and CES preferences; but this is inaccurate. In Melitz and Ottaviano (2008) gravity emerges while preferences are quasi-linear quadratic and Arkolakis et al. (2019) discuss how gravity equation emerges in the context of demand systems that feature a choke price. See also Head and Mayer (2014).

sector z country j, given by:

$$x_l(z) \equiv \left(\frac{c_l^*(z)}{c_l^{aut}(z)}\right)^{1+k(z)} \quad \text{and} \quad y_j(z) \equiv \frac{N_j^E(z)}{N_j^{E\ aut}(z)}.$$

As for the bundling parameters, introduce the followin definitions:

$$T_{jl}(z) \equiv \frac{\tau_{jl}(z)^{k(z)}}{L_l/L_j}\ ,\ K_{jl}(z) \equiv \left(\frac{c_l^{aut}(z)}{c_j^{aut}(z)}\right)^{1+k(z)}\ ,\ E_{jl}(z) \equiv \frac{f_j N_j^{E\ aut}(z)}{f_l N_l^{E\ aut}(z)} = \frac{L_j}{L_l}\ .$$

These definitions allow us collect in $T_{jl}(z)$ the trade costs in sector z from country j to country l weighted by relative market size, in $K_{jl}(z)$ the autarkic productivity cutoff of country j relative to country l in sector z and in $E_{jl}(z)$ the autarkic patterns of entry in country j relative to country l in sector z.

With this notation the structure of the equilibrium conditions can be highlighted as follows:

$$\begin{aligned}
\text{FEC**} &: \quad \sum_{l=1}^{J} \frac{K_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{1+k(z)} x_l(z) = 1 && \forall (j,z) \\
\text{OMC**} &: \quad \sum_{j=1}^{J} \frac{K_{jl}(z)E_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{k(z)} x_l(z) y_j(z) = 1 && \forall (l,z) \\
\text{LMC**} &: \quad \sum_{z=1}^{Z} \theta(z) y_j(z) = 1 && \forall j.
\end{aligned}$$

The equilibrium of the model consists of the solution of a system of $J + 2 \cdot J \cdot Z$ non-linear coupled equations in as many unknowns $\{w_j, x_j(z), y_j(z)\}$ for $j = 1, ..., J$ and $z = 1, ..., Z$. For a given vector of relative wages, FEC** is a linear system of $J \cdot Z$ equations in as many unknowns $x_j(z)$, which, substituted in OMC** yields a linear system of $J \cdot Z$ equations in as many unknowns $y_j(z)$. Therefore, the system of FEC** and OMC** determines a unique matrix of relative cutoff costs $x_j(z)$ and relative firm entry $y_j(z)$ for a given vector of relative wages.

Uniqueness. Taking the wage in country 1 as numeraire and rearranging the system of FEC** and OMC** within a sector z shows that the trade-induced change in country m's cost cutoffs $x_m(z)$ is increasing in its own relative wage w_m/w_1, and the trade-induced change in the measure of entrants $y_m(z)$ is decreasing in its own relative wage

w_m/w_1 for every sector:[4]

$$x_m(z) = 1 - \left(\frac{w_1}{w_m}\right)^{1+k(z)} \sum_{l \neq m}^{J} \frac{K_{ml}(z)}{T_{ml}(z)} \left(\frac{w_l}{w_1}\right)^{1+k(z)} x_l(z) \qquad \forall (m,z), \quad (24)$$

$$y_m(z) = \frac{1}{x_m(z)} - \left(\frac{w_m}{w_1}\right)^{k(z)} \sum_{j \neq m}^{J} \frac{K_{jm}(z)E_{jm}(z)}{T_{jm}(z)} \left(\frac{w_1}{w_j}\right)^{k(z)} y_j(z) \quad \forall (m,z). \quad (25)$$

The comparative statics in (24) and (25) corroborate the interpretation of the left hand side of LMC** as the country's aggregate labor demand function in open economy relative to autarky. Specifically, after substituting for $y_j(z)$ for $z = 1, ..., Z$ as implied by the system of FEC** and OMC**, the weighted sum of sectoral trade-induced changes in the numbers of entrants $\sum_{z=1}^{Z} \theta(z) y_j(z)$ is decreasing in the the country's own relative wage w_m/w_1 and increasing in the relative wage of other countries. It follows that, if an open economy equilibrium with diversification exists, then the monotonicity of the relative labor demand function in every country guarantees that the equilibrium is unique.[5]

Existence. For arbitrary parameter configurations for preferences, technologies, market sizes and trade costs, an open economy equilibrium with diversification is not granted. But, the equilibrium conditions FEC** and OMC** can be used to define a subset within the space of relative wage vectors $(1, w_2/w_1, ..., w_J/w_1) \in \Re_+^{J-1}$ that hosts (if nonempty) the equilibrium. An open economy equilibrium with diversification exists only if:

$$x_m(z) > 0 \iff \frac{w_m}{w_1} > [1 - x_m(z)]^{\frac{1}{1+k(z)}} > 0 \qquad \forall (m,z), \quad (26)$$

$$y_m(z) > 0 \iff \frac{w_m}{w_1} < [1 - x_m(z) y_m(z)]^{-\frac{1}{k(z)}} \qquad \forall (m,z). \quad (27)$$

After substitution of $\{x_k(z), y_k(z) : k = 1, ..., J\}$ as implied by FEC** and OMC**, conditions (26) and (27) define a finite number of interior sets $int\{\Omega_m(z)\}$ of compact sets $\Omega_m(z) \in \Re_+^{J-1}$, one for each sector $z = 1, ..., Z$. An equilibrium exists only if the inter-

[4]This conclusion holds not only locally (i.e. holding $\{x_l, y_j : l, j \neq m\}$ constant), but also globally as every x_k for $k \neq m$ is increasing in w_k/w_1 and decreasing in w_m/w_1 and every y_n for $n \neq m$ is decreasing in w_n/w_1 and increasing in w_m/w_1.

[5]Note that this argument is constructive: a numerical solution is obtained starting with a guess for the vector of relative wages and then augmenting the relative wage for countries with too much entry given the guess, i.e. $\sum_{z=1}^{Z} \theta(z) y_j(z) > 1$, and decreasing the relative wage for countries with not enough entry, i.e. $\sum_{z=1}^{Z} \theta(z) y_j(z) < 1$, till convergence.

section set $\Omega_m = \Omega_m(1) \cap ... \cap \Omega_m(Z)$ is nonempty for every country $m = 1, ..., J$. Thus, as FEC** and OMC** yield $(x_m(z), y_m(z))$ for all (m, z) as closed form expressions of model parameters and the vector of wages, the necessary condition for existence of an open economy equilibrium with diversification is:

$$0 < x_m(z) < 1 \text{ and } 0 < y_m(z) < 1/x_m(z) \quad \forall (m, z). \tag{28}$$

The system of $J - 1$ labor market clearing conditions LMC** is a continuous vector-valued and real-valued function defined on the compact set $\Omega = \Omega_1 \cap ... \cap \Omega_J$ mapping to the $(J-1)$-dimensional unit vector $\mathbf{1}$, i.e. $f : \Omega \to \Re_+^{J-1}$ such that $f(\omega) = 1$ for all $\omega \in \Omega \subset \Re_+^{J-1}$. If $\mathbf{1} \in \Omega$ held, then Brouwer Fixed-Point Theorem for $f : \Omega \to \Omega$ would imply that a solution to LMC** exists in Ω. Given that $w_m/w_1 = 1$ for all $m = 1, ..., J$ satisfies (26) and (27) for every country m and sector z, $\mathbf{1} \in \Omega$ actually holds. Hence, Brouwer Fixed-Point Theorem implies that (28) is a necessary and sufficient condition for existence of an open economy equilibrium with diversification.

In practice. An inspection of lower bounds defined in (26) and upper bounds defined in (27) suggests that Ω is not empty if trade costs are sufficiently high: everything else being the same, higher trade costs, through $T_{jm}(z)$ and $T_{ml}(z)$, decrease the lower bound and increase the upper bound of the feasible support for a relative wage in every country and sector.

Furthermore, an immediate test for existence of the equilibrium (sufficient but not necessary) consists of solving the FEC** and the OMC** given the same wage across countries $w_m/w_1 = 1$ for all $m = 1, ..., J$ and check if the solution satisfies, sequentially, first (26) and then (27). Once this test is passed then we know that an open equilibrium with diversification exists (at least the one prescribing the same wage in every country) and the LMC** condition can solved by iteration starting with the guess $w_m/w_1 = 1$ for all $m = 1, ..., J$ and updating the relative wage up for countries with too much entry and down otherwise.[6]

3.4 Discussion

Under the Pareto assumption the model presented our model belongs to the class of general equilibrium trade theories with monopolistic competition under free entry and

[6]Both procedures can be readily illustrated in the special case of symmetric countries, for which closed form solutions of the necessary and sufficient conditions are obtained. See appendix A.

additive-separable preferences discussed in Arkolakis et al. (2019), who extend previous work in Arkolakis et al. (2012) by relaxing the assumption of CES preferences.[7]

In this section we briefly discuss some related salient implications of trade in an open economy equilibrium with diversification. We start with gains from trade and then we look at trade-induced reallocations across sectors.

Gains from trade. The model predicts that in an equilibrium with diversification, i.e. $c_j^*(z) > 0$ and $N_j^E(z) > 0$ for every country j and sector z, there are gains from trade for every country and generated in every sector. This can be seen by rewriting FEC* as:

$$\left(\frac{c_j^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} = 1 - \sum_{l \neq j}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j}\frac{c_l^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} < 1 \quad \forall (j,z)\,, \tag{29}$$

that implies a lower cost cutoff in open economy $c_j^*(z) < c_j^{aut}(z)$ for every country j and sector z. Since welfare (19) is a geometric average of sectoral productivities, i.e. $1/c_j^*(z)$, welfare is predicted to be necessarily greater under trade than in autarky. Furthermore, despite tougher selection, in every sector the measure of varieties available (sourced anywhere) increases together with a lower cost cutoff:

$$N_l(z) = \frac{\gamma}{\alpha}\frac{\theta(z)}{(\mu_1(z) - \mu_2(z))\, c_l^*(z)}\,, \tag{30}$$

as it can be obtained by the system of output market clearing $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta(z) w_l$ and choke price $\hat{p}_l(z) = w_l c_l^*(z)$ evaluated under Pareto.[8]

Gains from trade are a classical result, that is customary in frameworks that feature a constrained-efficient equilibrium (such as with CES preferences). But in our setup the equilibrium is not constrained efficient. In particular, as discussed in Dhingra and Morrow (2019) for a closed or fully integrated economy, the way markups vary under our assumption on preferences implies that better-performing firms are too small and worse-performing firms are too large than in the social optimum. Hence, our setup shows that trade enhances efficiency even starting from a not constrained-efficient allocation.

[7] To see this, with reference to the group firms producing in country j sector z and selling to a destination l, call $v \equiv \hat{p}_l(z)/[\tau_{jl}(z)w_j c] = c_{jl}^*(z)/c \geq 1$ the measure of efficiency of a firm endowed with productivity $1/c$ relative to the other firms in the group. This change of variable makes our analysis in sections 2 and 3 isomorphic to the one in Arkolakis et al. (2019). Appendix B compares these setups in detail.

[8] See appendix A for a derivation of this result and other aggregate outcomes.

Trade-induced reallocations. Rewrite the sectoral version of labor market clearing LMC* as an *export equation*

$$N_j^E(z) \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} r_{jl}(c;z) dG_j(c;z) = \rho_j(z) w_j L_j \,,$$

and rewrite the output market clearing OMC* as an *import equation*

$$\sum_{m=1}^{J} N_m^E(z) \int_0^{c_{mj}^*(z)} r_{mj}(c;z) dG_m(c;z) = \theta(z) w_j L_j \,.$$

Accordingly, the ratio of total sales of country j in sector z (to itself and to the rest of the world) divided by total purchase of country j in sector z (from the world including the country itself) is given by $\rho_j(z)/\theta(z)$. It follows that a positive sectoral trade balance is characterized by a greater sectoral income share than expenditure share, $\rho_j(z) > \theta(z)$.

The system of a sector's labor market clearing condition LMC* and free entry condition FEC* yields the equilibrium relationship between the sector's employment share, $\rho_j(z)$, and the measure of entrants, $N_j^E(z)$, in open economy and in autarky. These are respectively determined as:

$$\frac{N_j^E(z)}{\delta(z)} = \rho_j(z) \frac{L_j}{f_j} \text{ and } \frac{N_j^{E\ aut}(z)}{\delta(z)} = \theta(z) \frac{L_j}{f_j}. \tag{31}$$

Sectoral profitability rate, i.e. $\delta(z)$, that is fixed by technological concentration $k(z)$ under Pareto, and market size relative to entry cost, i.e. L_j/f_j, are country and sector specific characteristics that do not vary by trade regime. Therefore, a positive sectoral trade balance in a given sector is associated with relatively more firm entry in open economy than in autarky:

$$\frac{N_j^E(z)}{N_j^{E\ aut}(z)} = \frac{\rho_j(z)}{\theta(z)} \,. \tag{32}$$

Average employment per entrant $\rho_j(z) L_j / N_j^E(z) = \theta(z) L_j / N_j^{E\ aut}(z)$ is not affected by trade, and this is true for average labor cost, revenue and profit per entrant. Sectors in which the country specializes (i.e. with a positive trade balance) grow unambiguously in terms of employment, sales, profit and measure of firms relative to import-competing sectors (i.e. with a negative trade balance).[9]

[9]This result also holds when comparing different levels of trade openness, with respect to sectors

If this result combines with tougher selection, then average employment, average revenue and average profit among incumbent firms increase. Furthermore, the measure of incumbent domestic firms unambiguously shrinks in import-competing sectors, both due to a lower measure of potential entrants and a tougher selection.[10]

At the country level, the entry of firms is bounded by market size and technological characteristics only, independently on the degree of trade openness. This can be seen by substituting FEC* in the aggregate LMC*, that yields an upper bound to the measure of entrants at the country level:

$$\sum_{z=1}^{Z} \frac{N_l^E(z)}{\delta(z)} = \sum_{z=1}^{Z} \frac{N_l^{E\ aut}(z)}{\delta(z)} = \frac{L_l}{f_l} \quad \forall l. \tag{33}$$

Therefore, if changes in trade openness lead to more entry in one sector, this must be compensated by less entry in other sectors.

All these considerations about trade-induced reallocation suggest that the ratio of sectoral income share $\rho_j(z)$ (which is endogenous) over sectoral expenditure share $\theta(z)$ (which is fixed under Pareto) can be considered as a model-based index of revealed comparative advantage.

4 Growth and welfare

How shall we think of growth in the model? Labor is the only factor of production. However, following a classical interpretation (in the spirit of Houthakker, 1955, Rybczynski, 1955, Solow, 1956, 1957 and Jones, 1971), there are also “missing factors" that is, country- and sector-specific resources complementary to labor, which are provided in fixed endowments, cannot be consumed, produced or used to finance entry, and cannot be traded.

Complementarity implies that an exogenous increase in the endowment of a country’s sector-specific resource determines a sector-biased outward shift of its production possibility frontier, which we will refer to as “growth”.

whose income shares expand (or shrink) in response to a trade shock.

[10]These effects are present in every sector when comparing trade with autarky, since cost cutoffs fall in all sectors. However, as we will discuss in the next section, if a shock other than a change in trade barriers hits an economy that is already open, then the changes in the cost cutoffs depend on the general equilibrium adjustment of wages

4.1 Growth as a resource shock

To translate growth in terms of the fundamentals of the model, recall that the exogenous technological coefficient $c_j^{max}(z)$ corresponds to the maximum units of labor per unit of output that a firm producing in sector z of country j operates with. The vector of these technological coefficients characterizes the smallest production possibility frontier of country j for a given endowment of labor L_j, defined as the locus of points $\{L_j/c_j^{max}(z) : z = 1, ..., Z\}$ were only the highest possible unit labor requirements to be used in all sectors. For this reason, in the wake of Eaton and Kortum (2002), we call $c_j^{max}(z)$ the "state of technology" of sector z in country j. We then define an exogenous growth shock in sector $s \in \{1, ..., Z\}$ of ('home') country $h \in \{1, ..., J\}$ as a sudden and permanent reduction in the exogenous labor requirement $c_h^{max}(s)$, while keeping all other exogenous characteristics of the economy unchanged. We will also refer to such shock as an improvement in the state of technology of sector s in country h.

To understand the consequences of such shock, let '0' and '1' label the equilibrium allocations before and after the shock respectively. Given the vector of relative wages before the shock, FEC** for sector s in country h can be rearranged as follows:

$$\sum_{l=1}^{J} a_{hl}^{0}(s) \left(\frac{x_l^1(s)}{x_l^0(s)} \right) = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)} \right)^{k(s)} < 1,$$

$$\text{where } a_{hl}^{0}(s) \equiv \frac{K_{hl}^{0}(s)}{T_{hl}^{0}(s)} \left(\frac{w_l^0}{w_h^0} \right)^{1+k(s)} x_l^0(s) \text{ and } \sum_{l=1}^{J} a_{hl}^{0}(s) = 1.$$

Inverting the implied linear system to solve for the changes in the cutoff costs yields:[11]

$$\begin{aligned} \left(\frac{c_h^{*1}(s)}{c_h^{*0}(s)} \right) &= \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)} \right)^{2k(s)} \\ &< \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)} \frac{c_l^{max\ 1}(s)}{c_l^{max\ 0}(s)} \right)^{k(s)} = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)} \right)^{k(s)} = \left(\frac{c_l^{*1}(s)}{c_l^{*0}(s)} \right) < 1. \end{aligned} \tag{34}$$

Therefore, for given pre-shock relative wages, sector s's post-shock equilibrium cutoff costs are lower in all countries, with the most pronounced fall in country h. Furthermore, when relative wages are held at their pre-shock values, the free entry conditions of all other sectors are not affected by the shock and no changes thus occur in their

[11]The derivations of this paragraph are reported in details in the appendix, Section (A.4).

cutoff costs in any country.

Now consider OMC** for sector s in country h. Given the vector of relative wages before the resource shock hits:

$$\sum_{j=1}^{J} b_{jh}^{0}(s)\frac{y_j^1(s)}{y_j^0(s)} = \frac{x_h^0(s)}{x_h^1(s)}\left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{k(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1$$

where $b_{jh}^{0}(s) \equiv \dfrac{K_{jh}^{0}(s)E_{jh}^{0}(s)}{T_{jh}^{0}(s)}\left(\dfrac{w_h^0}{w_j^0}\right)^{k(s)} x_h^0(s)y_j^0(s)$ and $\sum_{j=1}^{J} b_{jh}^{0}(s) = 1,$

and the second equality is implied by the FEC**. Inverting the associated linear system to solve for the changes in the measures of firms yields:

$$\frac{N_j^{E1}(s)}{N_j^{E0}(s)} = \frac{y_j^1(s)}{y_j^0(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1\ \forall j. \tag{35}$$

Therefore, in all countries, more firms are willing to enter the sector hit by the shock. Also in this case, when wages are held constant at their pre-shock values, output market clearing conditions in other sectors are not affected and thus their measures of entrants do not change.

Given that, at pre-shock wages, sector s's cutoff costs fall and its measures of entrants rise in all countries, but to a greater extent in country h, while they do not change in all other sectors, then $y_j^1(s) > y_j^0(s)$ and $y_j^1(z) = y_j^0(z)$ for all $z \neq s$ and all $j =\in \{1, ..., J\}$ imply that labor demand exceeds labor supply in all countries, but to a greater extent in country h. Hence, as all countries' labor supplies are exogenously fixed, to restore market clearing wages have to increase everywhere, but to a greater extent in country h. Higher wages increase the cutoff costs and decrease the measure of entrants. For sector s, they thus dampen the fall in the cutoff cost and the rise measure of entrants with respect to their pre-shock values. For all other sectors, they lead to larger cutoffs and smaller measures of entrants with respect to the pre-shock equilibrium. The more a country's wage increases relative to the other countries, the more its cutoffs rise and the measures of its firms fall in the sectors that are not hit by the shock.

4.2 Extended "welfare formula"

The previous section has clarified how a resource shock propagates in the general equilibrium of the multi-country multi-sector economy: at least in some sector in every country higher relative wages lead to higher cutoff costs and thus higher choke prices. This raises concerns about the possibility of net welfare losses, which can be readily addressed by noticing that the model generates a handy "welfare formula" in the wake of Arkolakis et al. (2012) and Arkolakis et al. (2019). The formula provides a convenient way to summarize the prediction of the model for welfare in response to trade shocks as in those paper, but also to other shocks, such as a resource shock.

To see this, consider value added $w_j L_j$ in country j and recall that $\theta(z) w_j L_j$ is the expenditure of country j on goods of sector z. A fraction of this expenditure is allocated to domestic production, and to characterize this fraction we define country j's domestic trade share in sector z as $\lambda_{jj}(z) \equiv X_{jj}(z)/[\theta(z) w_j L_j]$, where domestic sales in the domestic market are given by

$$X_{jj}(z) = \underbrace{N_j^E(z)[w_j c_j^{max}(z)]^{-k(z)} \hat{p}_j(z)^{k(z)}}_{\text{measure of firms}} \underbrace{\zeta_X(z) \hat{p}_j(z) L_j}_{\text{average firm sales}} .$$

Substituting for the choke price $\hat{p}_j(z) = w_j c_j^*(z)$, and for the measure of entrants $N_j^E(z)$ as implied by the system of free entry and sectoral labor market clearing conditions $\delta(z)\rho_j(z) w_j L_j = f_j w_j N_j^E(z)$ yields the following expression for the cutoff cost in terms of the sectoral domestic trade share $\lambda_{jj}(z)$ and sectoral employment share $\rho_j(z)$:

$$c_j^*(z)^{1+k(z)} = \frac{\theta(z) c_j^{max}(z)^{k(z)}}{\zeta_X(z)\delta(z)} \frac{f_j}{L_j} \frac{\lambda_{jj}(z)}{\rho_j(z)}.$$

Further substitution for the cutoff cost in (19) allows us to evaluate welfare through a formula that recalls the one based on CES preferences in Arkolakis et al. (2012):

$$W_j = \prod_{z=1}^{Z} B(z) \left(\frac{c_j^{max}(z)^{k(z)}}{L_j/f_j} \frac{\lambda_{jj}(z)}{\rho_j(z)} \right)^{-\frac{\beta(z)}{1+k(z)}} \tag{36}$$

where $B(z) \equiv \beta(z)^{\beta(z)} \left(\zeta_X(z)\delta(z)/\theta(z)\right)^{\frac{\beta(z)}{1+k(z)}}$ is a constant bundle of sector-specific taste parameters and technological concentration. It follows that the only endogenous outcomes needed to evaluate the sectoral cutoff costs and thus welfare are the sectoral domestic trade shares $\lambda_{jj}(z)$ and the sectoral employment shares $\rho_j(z)$. Arkolakis et al. (2019) pointed out that this should be the case for a class of models that ours belongs to, so this conclusion confirms their result. However, differently from them, we allow technological concentration to vary across sectors, which is crucial for understanding the welfare effect of the resource shock we study, but also for gaining deeper insights on the welfare effects of the trade shocks they focus on.[12]

4.3 Incomplete "growth pass-through"

Expression (36) is the analogue in our framework of equation (25) in Melitz and Redding (2013), who characterize the equilibrium sectoral productivity cutoffs in the context of multi-country, multi-sector trade models with monopolistic competition for CES preferences across varieties.[13]

Comparison with Melitz and Redding (2013) sheds light on a key difference from Arkolakis et al. (2012) not considered in Arkolakis et al. (2019). While with CES preferences a change in the upper bound of the support for the cost distribution that holds the ratio of sectoral domestic trade shares and employment share constant is fully passed on to the equilibrium cutoff costs, the pass-through is incomplete in the presence of a choke price:

$$\left.\frac{\partial \log c_j^*(z)}{\partial \log c_j^{max}(z)}\right|_{\frac{\lambda_{jj}(z)}{\rho_j(z)}} = \begin{cases} \frac{k(z)}{1+k(z)} \in (0,1) & \text{with a choke price} \\ 1 & \text{with CES preferences} \end{cases} . \tag{37}$$

The reason for this different behavior is that, while with CES preferences without a choke price, the cutoff cost in the destination market does not matter for average firm sales, with a choke price in the destination market average firm sales are proportional to the local cutoff cost.

This means that, in the presence of a choke price, if the average cost in sector z of country j decreases through an exogenous reduction in the upper bound $c_j^{max}(z)$ and we do not observe any change in the ratio of domestic trade share $\lambda_{jj}(z)$ over employment

[12] In the appendix, Section B.3 shows that the effect a foreign trade shock is smaller due to a lower cost pass-through on average (which is the point of Arkolakis et al., 2019), but also in sectors characterized by lower technological concentration that matter relatively less due to a comparatively smaller expenditure share, $\theta(z)$. Clearly this differential effect vanishes if differences in sectoral concentrations are not taken into account so that that $\theta(z) \equiv \beta(z)$ holds.

[13] With respect to their notation, we have the equilibrium cost cutoff equal to the inverse of the equilibrium productivity cutoff $c_j^*(z)^{1+k(z)} \equiv 1/(\varphi_{jjz}^{\star})^{k(z)}$, the upper bound of the cost support equal to the lower bound of productivity support augmented with fixed market access cost $c_j^{max}(z)^{k(z)} \equiv 1/[f_{jjz}\varphi_{min}^{k(z)}]$, a fixed cost of entry $f_j \equiv f_{Ejz}$ (which could be made sector specific also in our setup with no loss of tractability), sectoral consumption shares $\theta(z) \equiv \beta_z$, a profitability index $\zeta_X(z)\delta(z) \equiv \frac{k(z)-(\sigma_z-1)}{(\sigma_z-1)}$, endogenous sectoral employment shares defined as $\rho_j(z) \equiv L_{jz}/\bar{L}_j$, and endogenous domestic trade share defined as $\lambda_{jj}(z) \equiv \lambda_{jjz}$.

share $\rho_j(z)$, then average firm sales fall less than proportionately and the remaining adjustment takes place through more firm entry. Furthermore, the entry of firms is more pronounced in sectors with lower technological concentration. In contrast, with CES preferences the model would not predict any adjustment in either the intensive or the extensive margin and, therefore, also asymmetries across sectors in technological concentration would not play a role.

4.4 Welfare response to a resource shock

The incomplete growth pass-through documented in (37) has two implications. First, observed changes in domestic trade shares and employment shares are translated into welfare changes at a discount rate $-[1+k(z)]$ that is larger in absolute value than the trade elasticity $-k(z)$. Second, sectors with lower technological concentration contribute proportionally less to welfare changes, as implied by the fact that the proportional change from $k(z)$ to $1+k(z)$ declines as $k(z)$ grows. Hence, a sector's contribution to welfare change is attenuated the more its pass-through is incomplete.

Rewriting welfare (36) to emphasize the pass-through rate $k(z)/[1+k(z)]$ yields

$$W_j = \prod_{z=1}^{Z} B(z) \left(\frac{f_j c_j^{max}(z)^{k(z)}}{L_j} \frac{\lambda_{jj}(z)}{\rho_j(z)} \right)^{-\frac{\beta(z)}{k(z)} \frac{k(z)}{1+k(z)}} \tag{38}$$

which converges to the formula in Arkolakis et al. (2012) as the pass-through becomes increasingly complete (i.e. $k(z)/[1+k(z)] \rightarrow 1$). Otherwise, expression (38) offers a parametrization of the results discussed (on average) in Arkolakis et al. (2019) that remains highly tractable despite allowing for sectoral heterogeneity in the degree of technological concentration.

In this respect, it is instructive to further contrast our framework characterized by incomplete pass-through, in the sense of (37), with its analogue under CES. The goal here is not to assess the impact of any specific resource shock, but rather to highlight what (38) and its CES analogue imply for our understanding of welfare changes when they are calibrated on the same observed outcomes (i.e. on the same historical data on sectoral domestic trade shares and employment shares) and as on the same exogenous variation.

Let h indicate again the home country. Given welfare (38) and computing percentage changes in log-difference before and after the shock as $\Delta \ln x = \ln x^1 - \ln x^0$ yields

$$\Delta \ln W_h =$$
$$= - \sum_{z=1}^{Z} \beta(z) \Delta \ln(c_h^*(z))$$
$$= - \sum_{z=1}^{Z} \frac{k(z)}{1+k(z)} \frac{\beta(z)}{k(z)} \left[\Delta \ln f_h - \Delta \ln L_h + \Delta \ln \lambda_{hh}(z) - \Delta \ln \rho_h(z) + k(z) \Delta \ln c_h^{max}(z)\right].$$

Consider a shock that hits sector s only in country h (i.e. $\Delta \ln c_h^{max}(s) < 0$ and $\Delta \ln c_h^{max}(z) = 0$ for every $z \neq s$) while keeping all other exogenous parameters (endowments, entry costs, trade costs, technological concentration and preferences) unchanged everywhere. The impact of this shock is given by:

$$\Delta \ln W_h^{IPT} = \qquad (39)$$
$$= \underbrace{\sum_{z=1}^{Z} \frac{k(z)}{1+k(z)} \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right]}_{\text{GE trade effect}} - \underbrace{\frac{k(s)}{1+k(s)} \beta(s) \Delta \ln c_h^{max}(s)}_{\text{direct resource effect}},$$

where we emphasize that the formula accounts for incomplete pass-through (IPT). Crucially, the shock is designed to work in a controlled environment, with the same interpretation of the "ex-post" result in Arkolakis et al. (2012). Hence, changes in observed outcomes (sectoral domestic trade shares and employment shares) are attributed, by design, to the resource shock. Only under these circumstances, the general equilibrium (GE) effect of the resource shock can be identified and disentangled from the direct effect.

Under these premises, if a model equivalent to ours, but with CES preferences across varieties, were calibrated on the same shock and observed outcomes, it would compute a change in welfare equal to:[14]

$$\Delta \ln W_h^{CES} = \sum_{z=1}^{Z} \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right] - \beta(s) \Delta \ln c_h^{max}(s). \qquad (40)$$

A comparison between expression (39) and (40) shows that, once calibrated on the same data, the two models' assessments of the welfare changes differ only because of the

[14]We do not report the derivations, since we refer to the canonical multi-country, multi-sector, quantitative trade model with heterogeneous firms, free entry and monopolistic competition, discussed in Arkolakis et al. (2012) and in the handbook chapter by Melitz and Redding (2013).

incomplete pass-through featuring in the IPT expression, which scales down both the GE trade effect and the direct resource effect relative to the CES expression.

4.5 Hunting fantastic beasts

In the Introduction we have defined a "fantastic beast" as a magical creature that cannot be seen unless a wizard searches for it, but that plays a significant role in the real world.

Following this metaphor, we now want to search for welfare changes induced by a resource shock that we fail to see when we look at the data through a CES lens. A shock like this is defined by inverting (40) for $\Delta \ln W_h^{CES} = 0$ to obtain

$$\beta(s)\Delta \ln c_h^{max}(s) = \sum_{z=1}^{Z} \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right] \tag{41}$$

which we can substitute in the wizard's world that is visible through the IPT lens:

$$\Delta \ln W_h^{IPT} = \sum_{z=1}^{Z} \left(\frac{k(z)}{1+k(z)} - \frac{k(s)}{1+k(s)}\right) \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right]. \tag{42}$$

The necessary conditions to find fantastic beasts, that is, to see welfare changes that invisible through a CES lens, are:

1. **Multi-sector economy**: the contribution of the sector hit by a resource shock in explaining why welfare responses under IPT deviate from those under CES is null; therefore, in a one-sector economy $\Delta \ln W_h^{CES} = 0$ implies $\Delta \ln W_h^{IPT} = 0$.

2. **Open economy**: in closed economy the domestic trade share equals one by construction, hence $\Delta \ln \lambda_{hh}(z) = 0$ holds, and the employment share is fixed such that $\rho_h(z) = \theta(z)$, hence $\Delta \ln \rho_h(z) = 0$ holds; therefore, the GE effect of a resource shock does not operate, only the direct resource effect is at work and it implies proportional welfare changes $\Delta \ln W_h^{IPT} = \frac{k(s)}{1+k(s)} W_h^{CES}$.

3. **Heterogeneity in technological concentration across sectors**: if sectors have the same sectoral concentration (i.e., $k(s) = k(z) = k$), then welfare responses under IPT do not deviate from those under CES; therefore, if there is no heterogeneity in technological concentration across sectors, then $\Delta \ln W_h^{CES} = 0$ implies $\Delta \ln W_h^{IPT} = 0$ and, more generally, $\Delta \ln W_h^{IPT} = \frac{k}{1+k} W_h^{CES}$, which directly speaks to the result in Arkolakis et al. (2019).

It follows that, only because we have modelled a multi-sector open economy with heterogeneous technological concentration across sectors, we can hope to see some fantastic beasts. To find them, we take historical data on the sectoral employment shares and domestic trade shares of the home country, split sectors in two groups according to the signs of their contribution to welfare changes and introduce the following notation:

$z^h_{(-)}$ refers to country h's sectors where the ratio $\rho_h(z)/\lambda_{hh}(z)$ decreases after the shock, such that $[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)] < 0$; these are "bad" sectors with a negative contribution to the country's welfare.

$z^h_{(+)}$ refers to country h's sectors where the ratio $\rho_h(z)/\lambda_{hh}(z)$ increases after the shock, such that $[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)] > 0$; these are "good" sectors with a positive contribution to the country's welfare.

This grouping is country specific and has to be interacted with heterogeneity in sector-specific technological concentration to understand whether fantastic beasts exist that may inflict paradoxical welfare losses on country h in response to a positive resource shock ("immiserizing growth") or gift the country welfare gains in response to a negative resource shock ("enriching decline") despite the prediction of no welfare change under CES. Specifically, consider a positive CES-neutral resource shock in sector s of country h such that $\Delta \ln c^{max}_h(s) < 0$ and (41) holds. Then, a sufficient (though not necessary) condition for immiserizing growth in the country is that the following requirements are both met and at least one of them is met strictly:

(i) $k(s) \leq \min_{z \in z^h_{(-)}} \{k(z)\}$, i.e. technology is less concentrated in the sector hit by the resource shock than in the "bad" sectors of the country;

(ii) $k(s) \geq \max_{z \in z^h_{(+)}} \{k(z)\}$, i.e. technology is more concentrated in the sector hit by the resource shock than in the "good" sectors of the country.

If these requirements were met with opposite signs, and at least one were met strictly, then we would have a sufficient (though not necessary) condition for 'enriching decline'.

5 Theory with numbers

As proof of concept, we now set out to show see whether any fantastic beasts can be identified in real-world data, that is, whether in an IPT setup there is any scope for welfare-reducing positive resource shocks ("immiserizing growth") or welfare-improving negative resource shocks ("enriching decline") that would be welfare-neutral in a CES one.

Consider an unanticipated and permanent increase in resources specific to the production of sector s in home country h, while keeping all other exogenous parameters (i.e. domestic and foreign economic fundamentals) constant. As previously discussed, such shock takes the form of an improvement in the country- and sector-specific state of technology. Formally, let $\boldsymbol{L}$, $\boldsymbol{f}$, $\boldsymbol{\tau}$ and $\boldsymbol{c}$ be the vectors of parameters respectively capturing market sizes, fixed costs, trade costs and the upper bound of the support of the technological coefficients for all countries and sectors. Then we have:

Definition. *A 'CES-neutral' domestic resource shock in country h sector s is a change from* $c_h^{max}(s)$ *to* $c_h^{max}(s)' \neq c_h^{max}(s)$ *such that* $\boldsymbol{L}' = \boldsymbol{L}$, $\boldsymbol{f}' = \boldsymbol{f}$, $\boldsymbol{\tau}' = \boldsymbol{\tau}$, $c_j^{max}(z)' = c_j^{max}(z)$ *for all countries* $j \neq h$ *sectors* $z = 1, ..., Z$, $c_h^{max}(z)' = c_h^{max}(z)$ *for all sectors* $z \neq s$ *and* $\Delta \ln c_h^{max}(s) = c_h^{max}(s)' - c_h^{max}(s)$ *satisfies condition* (41).

According to this condition, computing a CES-neutral domestic resource shock in sector s of country h requires trade and production data on the sectoral domestic trade shares $\lambda_{hh}(z)$ and employment shares $\rho_h(z)$, in addition to estimates of the sectoral Cobb-Douglas consumption shares $\beta(z)$ and trade elasticities $k(z)$.[15]

Before looking at the data, we pause to remark that, although a resource shock is different from a trade shock, a CES-neutral domestic resource shock can only exist in open economy. In this respect, the consequences of a CES-neutral resource shock belong to the welfare responses that are channeled through trade.[16]

[15] We proceed along same lines of the sufficient statistic approach by Arkolakis et al. (2012). This is not immune to criticism. In particular, as pointed out by Melitz and Redding (2015), when using endogenous outcomes as sufficient statistics, one needs to assume that no change occurs in the structural parameters of the model across countries and over time. This is less of a concern in our case as our goal of our proof of concept is not to predict welfare changes, but rather to compare two different model-specific computations of welfare changes, $\Delta \ln W_h^{IPT}$ and $\Delta \ln W_h^{CES}$, for identical values of endogenous outcomes and structural parameters.

[16] The peculiarity of a resource shock is that it directly affects a country and a sector in isolation from other countries and sectors (see, e.g., Pelzl and Poelhekke (2021), or Caliendo et al. (2018) within the quantitative trade literature). This 'local' feature makes the resource shock fundamentally different from a perturbation to other parameters shaping the equilibrium of the model. For example, changes in trade costs $\tau_{jl}(z)$ affect sector-specific but bilateral parameters, with more than one country involved; changes in market sizes L_j and entry costs f_j concern country-specific parameters that affect all sectors; changes in technological concentration $k(z)$, which impact on expenditure shares $\theta(z)$ and profitability $\delta(z)$, or change in tastes that influence $\beta(z)$, are all about sector-specific parameters, but hit all countries simultaneously.

5.1 Data and sources

The main data source we use is the Trade in Value-Added database (TiVA 2023) by the OECD. It provides information on production, consumption, international trade and global economic integration based on Inter-Country Input-Output (ICIO) tables. The data is available annually for the period 1995-2020, for 76 countries (including all OECD countries and the rest of the world) and 45 industries classified by economic activity. Among these, we consider only manufacturing, which corresponds to 17 sectors in the TiVA industry classification.

We use data on consumption in value-added ($CONS\_VA$) by source country and sector, and value added ($VALU$) by origin, destination and sector to compute: $\beta(z)$ as the cross-country average expenditure shares at the sector level; $\lambda_{jj}(z)$ as the domestic consumption share in value-added at country-sector level; and the sectoral employment share $\rho_j(z)$ as the share of value-added at country-sector level.

The key parameter of our analysis is the concentration of the distribution of technologies by sector $k(z)$. The structural interpretation of the gravity equation we have provided implies that $k(z)$ corresponds to the trade elasticity. We can, therefore, use existing estimates of the trade elasticity at TiVA industry-level that are available in the literature, relying specifically on those provided by Fontagné et al. (2022).[17]

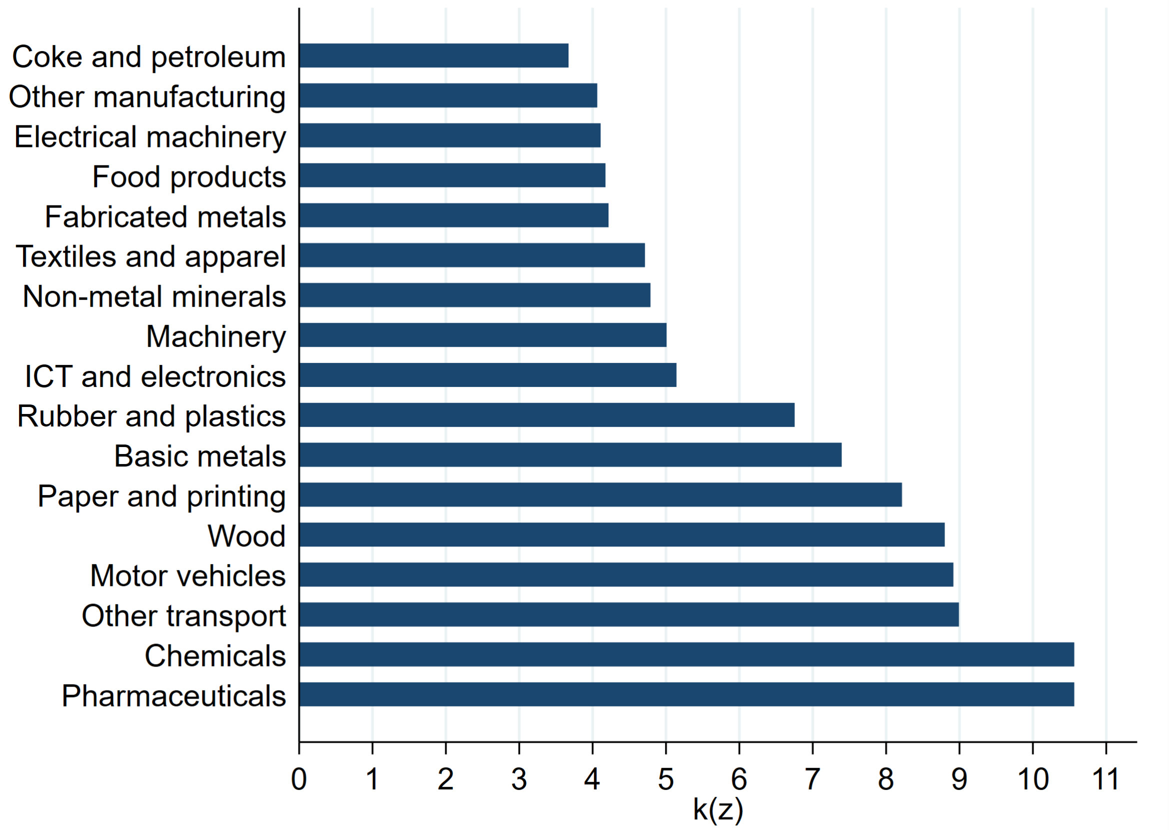
5.2 Preliminary evidence

In light of the discussion in Section 5.5, to find fantastic beasts in a real-world multi-sector open economy it is necessary that there is variation in the technological concentration across sectors as well as in the sectoral domestic trade shares, employment shares and their ratios within countries over time.

We start by looking at the variation of the technological concentration parameter. Figure 1 reports the estimated $k(z)$ across manufacturing industries. Sector C-19 *Coke and refined petroleum product* is the one with the lowest concentration, $k(z) = 3.67$; sectors C-21 *Pharmaceuticals products* and C-20 *Chemical products* exhibit the highest concentration, $k(z) = 10.56$.[18] Accordingly, based on their concentration parameters, a

[17]We have chosen this source because the estimates of the trade elasticity are computed at the level of TiVA sectors and are thus consistent with the rest of our analysis. For a comparison with other estimates in the literature and an assessment of robustness, we refer the interested reader to Fontagné et al. (2022).

[18]Recall that higher concentration implies that upon entry a firm is more likely to draw a unit labor requirement closer to the upper bound of the support, and it is therefore more likely to subsequently



Note: sectors are the manufacturing industries in TiVA classification; estimates of the parameter $k(z)$ are from Fontagné et al. (2022).

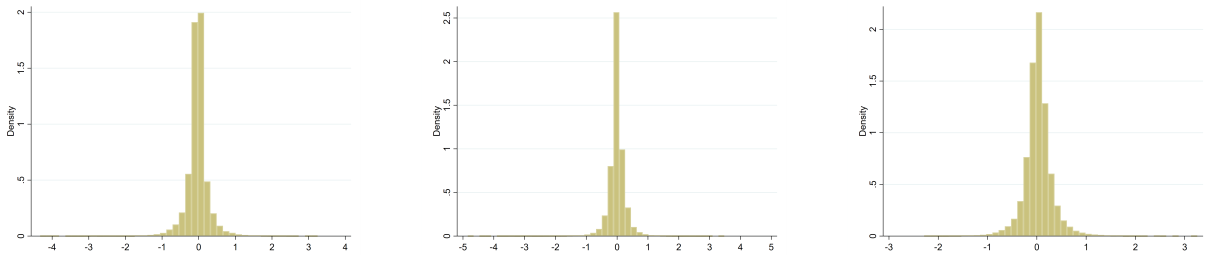
Figure 1: Estimates of the parameter of technological concentration by sector

resource shock in the petroleum industry is associated with a growth pass-through of $78,59\%$, while a resource shock in the pharmaceutical or chemical industries is associated with a growth pass-through of $91,35\%$.

We turn next to the variation over time in the sectoral domestic trade shares, the employment shares and their ratios at the country level. Using data from all 76 countries, all 17 manufacturing industries and all years from 2000 to 2020, we compute 1-year, 3-year and 5-year changes in log difference of country-specific sectoral domestic trade shares, sectoral employment shares and their balance. The three panels in Figure 2 report the results for the three statistics referred to the 5-year change; the 1-year and 3-year changes exhibit similar patterns. In both the domestic trade shares and the em-

leave the market without producing. Higher concentration also means that, among firms that actually produce, there is a larger share of small firms that have much higher labor unit input requirements than those of the much fewer most efficient and largest firms in the sector.

ployment shares there is substantial variation around zero, which is suggestive of a stationary data generating process in log differences. Furthermore, while some sectors expand while other sectors shrink, the employment shares do not change proportionally with the domestic trade shares.



$\Delta \ln \lambda_{jj}(z)$ $\Delta \ln \rho_j(z)$ $\Delta \ln \rho_j(z) - \Delta \ln \lambda_{jj}(z)$

Figure 2: Change in domestic trade shares and employment shares

Based on these premises, it is now possible to apply equation (41) to compute the CES-neutral resource shock in sector s of country h and then evaluate the corresponding IPT welfare change through expression (42).[19] Table 1 reports the summary statistics for the computed CES-neutral resource shock $\Delta \ln c_h^{max}(s)$. The mean of the distribution is 2.3% over a 1-year interval, 7.4% over a 3-year interval and 12.6% over a 5-year interval. While the mean is positive, there is enough variation to allow for both shocks that improve the state of technology ($\Delta \ln c_h^{max}(s) < 0$) and shocks that worsen it ($\Delta \ln c_h^{max}(s) > 0$). This feature comes from the the data rather than by construction and makes it feasible to look for immiserizing growth as well as enriching decline.

Table 1: Summary statistics on CES-neutral resource shock

	mean	std. dev.	p10	p50	p90
1-year % change	2.310	36.347	-25.620	2.027	31.675
3-year % change	7.416	56.648	-39.029	5.513	57.068
5-year % change	12.581	70.299	-44.904	8.932	74.473

Before doing that, however, it would be reassuring to know that the computed CES-neutral resource shocks are in the same ballpark as actual resource shocks documented in the literature. As a recent example falling in our period of observation, Caliendo et al.

[19] It should be emphasized that we are not assuming that the shock affecting the data generating process is a resource shock. Several shocks (e.g. foreign shocks as in Arkolakis et al., 2012) might well hit the economy, and thus drive the observed changes of the domestic trade shares and employment shares. However, the reasons why these shares change is immaterial for the computation of the CES-neutral resource shock.

(2018) document a shale oil boom of 9% TFP growth in North Dakota over the period from 2007 to 2012 and a 14.6% TFP growth in the industry of Computers and Electronics in California during the same period. Hence, our CES-neutral resource shocks are broadly in line with actual technological shocks reported in other studies.

5.3 Finding fantastic beasts

Expressions (41) and (42) makes clear that the exact values of CES-neutral $\Delta \ln c_h^{max}(s)$ and its impact on $\Delta \ln W_h^{IPT}$ depend on which sector s in which country h those expressions are applied to. Moreover, while the sign of the CES-neutral shock is not sector specific, the sign of a country's welfare response depends on the technological concentration of the sector hit by the shock relative to all other sectors.

To highlight the importance of the sectoral choice, we first consider the US case. While we focus on 5-year percentage changes (which we consider as the most conservative option), the conclusions are all confirmed also using 1-year and 3-year changes. In the 21 years from 2000 to 2020, the CES-neutral domestic resource shock over 5 years is found to be expansionary only four times: in year 2013 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.05\%$, year 2016 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.005\%$, year 2019 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.18\%$, and year 2020 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.7\%$.

For the sake of argument, we choose the sector s to be shocked based on observed resource shocks. In particular, we refer to Caliendo et al. (2018) who document resource shocks for two sectors: *Coke and petroleum* (henceforth, simply 'oil'), a sector with the lowest concentration $k(oil) = 3.67$ and a consumption share $\beta(oil) = 5.90\%$; and *ICT and electronics* (henceforth, simply 'ICT'), a sector with a rather median concentration $k(ict) = 5.16$ and a consumption share $\beta(ict) = 6.75\%$. Plugging these numbers in expression (41), we calculate a positive CES-neutral resource shock for the US oil sector during 2009-2013 equal to $\Delta \ln c_{US}^{max}(oil) = -0.05\%/\beta(oil) = -0.85\%$. By expression (42), the corresponding change in US welfare amounts to $\Delta \ln W_{USA}^{IPT} = +0.015\%$. Hence, in the time period under consideration, the elasticity of US welfare to the domestic CES-neutral domestic resource shock in the oil *oil* sector is 0.020. Analogously, we can calculate a positive CES-neutral resource shock for the US ICT sector for the same period, which evaluates to $\Delta \ln c_{USA}^{max}(ict) = -0.05\%/\beta(ict) = -0.74\%$. The corresponding welfare change is $\Delta \ln W_{USA}^{IPT} = +0.018\%$, with the elasticity of welfare to the shock equal to 0.025. In both cases, the IPT setup captures welfare gains that do not arise under CES.

Having highlighted the importance of the sectoral dimension, we can extend the analysis to all 76 countries, 17 sectors and years in the dataset. We again focus on 5-year time intervals, with 1-year and 3-year intervals generating similar results.

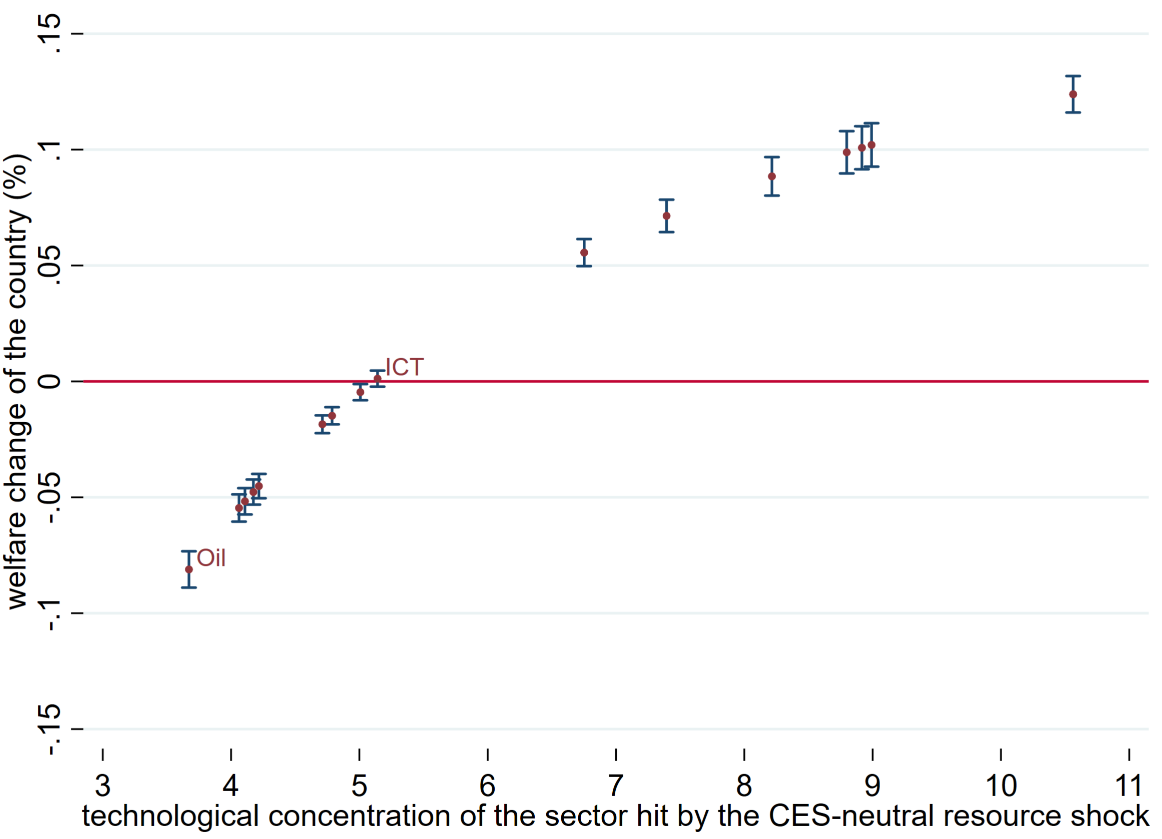


Figure 3: Welfare response to expansionary resource shocks

Pooling all countries, Figure 3 depicts how welfare changes in response to an expansionary CES-neutral domestic resource shock ($\Delta \ln c_{US}^{max} < 0$). Each dot refers to a sector. Its vertical coordinate corresponds to the mean point estimate of the impact on country's welfare given that the shock hits a given sector whose technological concentration is reported on the horizontal axis in ascending order, as in Figure (1). Around the mean, the figure also reports the 95% confidence intervals, based on variation across countries and years. The figure shows that immiserizing growth materializes when an expansionary CES-neutral resource shock hits a sector characterized by relatively low technological concentration. In contrast, when the shock hits a sector with high technological concentration, the IPT welfare-change is positive.

Symmetric results are portrayed in Figure 4 for contractionary CES-neutral domestic resource shocks. Mean point estimates are again precisely estimated, and reveal

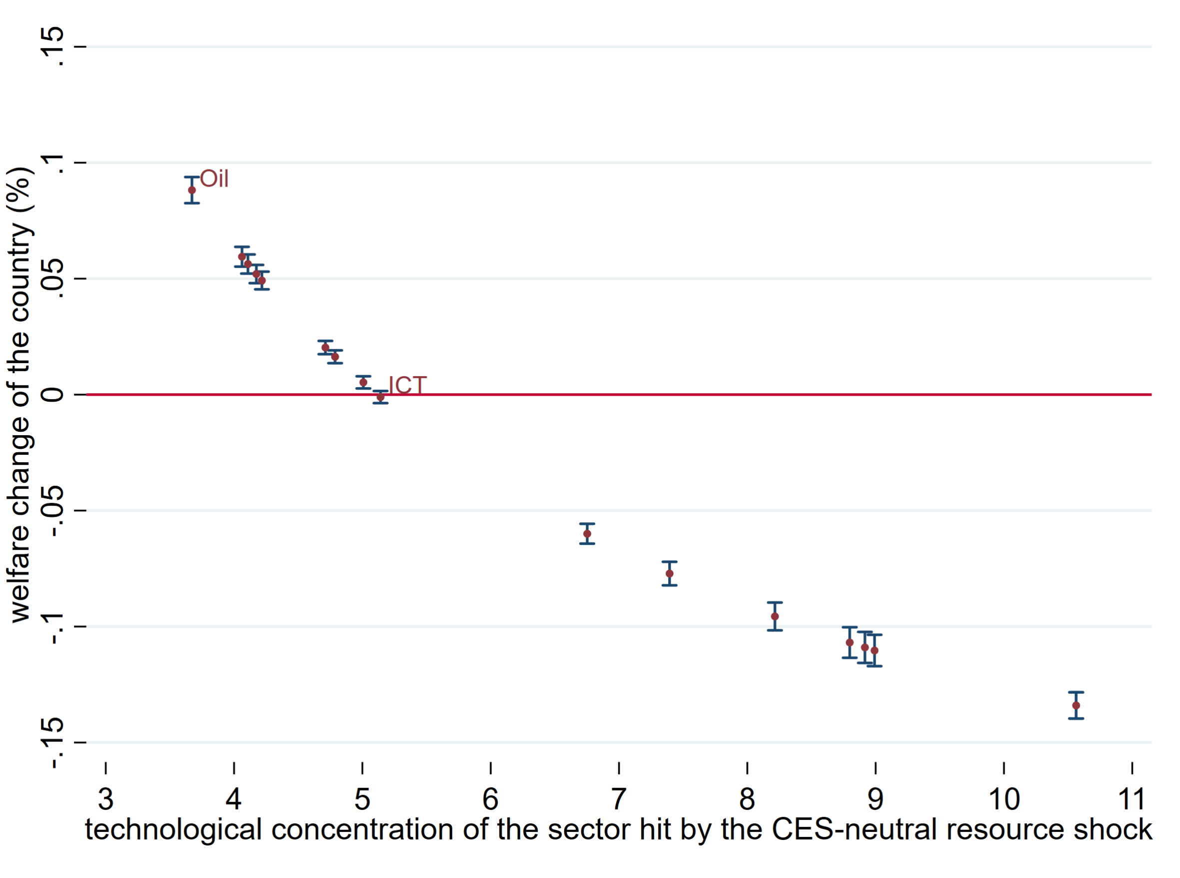


Figure 4: Welfare response to contractionary resource shocks

enriching decline when the shock hits sectors characterized by low concentration.

In both figures we keep track of the two sectors discussed in the US case: Oil and ICT. The first sector is the typical sector considered in the literature on the "Dutch disease" as the cause of immiserizing growth. As this is the sector with the lowest technological concentration and thus the most incomplete pass-through, our model provides new insights on the classical result based on monopolistic competition, firm heterogeneity and markup distortions. The second sector is important for recent growth episodes, as suggested by Caliendo et al. (2018). Interestingly, in the two figures ICT appears to be the sector associated with the concentration threshold below which immizerizing growth or enriching decline materialize. In other words, a CES-neutral resource shock in ICT is indeed neutral also from welfare viewpoint.

Figures 5 and 6 repeat the exercise across 15 selected countries. For each country, Figure 5 depicts the welfare change in response to an expansionary CES-neutral resource shock against technological concentration, with 95% confidence intervals relying here on time variation only. The conclusion that we have reached on the pooled anal-

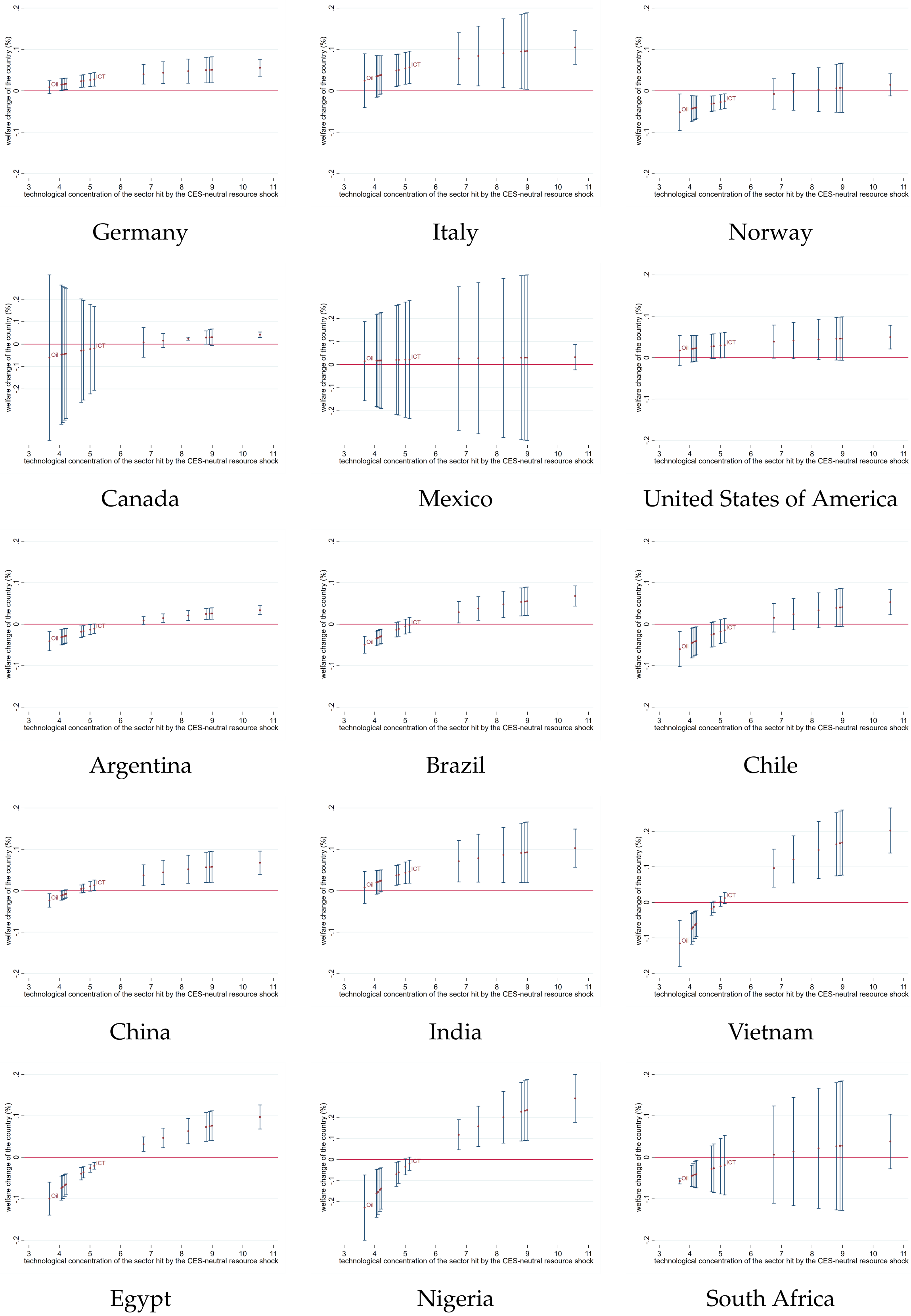


Figure 5: Welfare responses to expansionary resource shocks, in selected economies

ysis finds confirmation in the overall association of lower concentration with smaller positive or more negative welfare changes. There is, however, a lot of cross-country variation due to different sectoral specialization. While the evidence of immiserizing growth is not statistically significant everywhere, in all significant cases it materializes when the shock hits sectors with lower technological concentration, in particular the oil sector.

When interpreting the magnitude of the effects, one should consider that, on average, each sector weighs less than 6% in consumers' expenditure. At the median magnitude of a CES-neutral shock (i.e. 8.9%), even if the shock were entirely transmitted to welfare in proportion with the consumption share (i.e., there were no general equilibrium feedback effects and pass-through were complete), the response would have be smaller than 0.5%. Thus, to help comparison across countries, the scale of the vertical axis is set in the range ±0.2% everywhere.

Although the pattern of an increasing average relationship is common, the range of sectors conducive to immizerizing growth if hit by a CES-neutral expansionary shock varies substantially. For example, a shock hitting ICT implies statistically significant welfare losses in three countries (Norway, Argentina and Egypt) and statistically significant welfare gains in five countries (Germany, Italy, USA, China and India).

Figure 6 considers a contractionary s CES-neutral resource shock. There is a robust negative relationship between technological concentration and welfare changes everywhere, with statistically significant evidence of enriching decline when the shock hits sectors with lower technological concentration.

6 Conclusion

We have developed a quantitative trade model with incomplete constant absolute pass-through (IPT) that can predict both "immiserizing growth"and "enriching decline"when standard models featuring CES demand and thus complete pass-through predict neither. In the former case, a domestic resource increase that does not change welfare under CES leads to lower welfare under IPT. In the latter case, a domestic resource reduction that does not change welfare under CES leads to higher welfare under IPT.

We have shown that the reason for these divergences is that IPT allows for richer reallocation patterns between firms and sectors than CES does. We have argued that constant absolute pass-through is not essential. Nevertheless, together with the as-

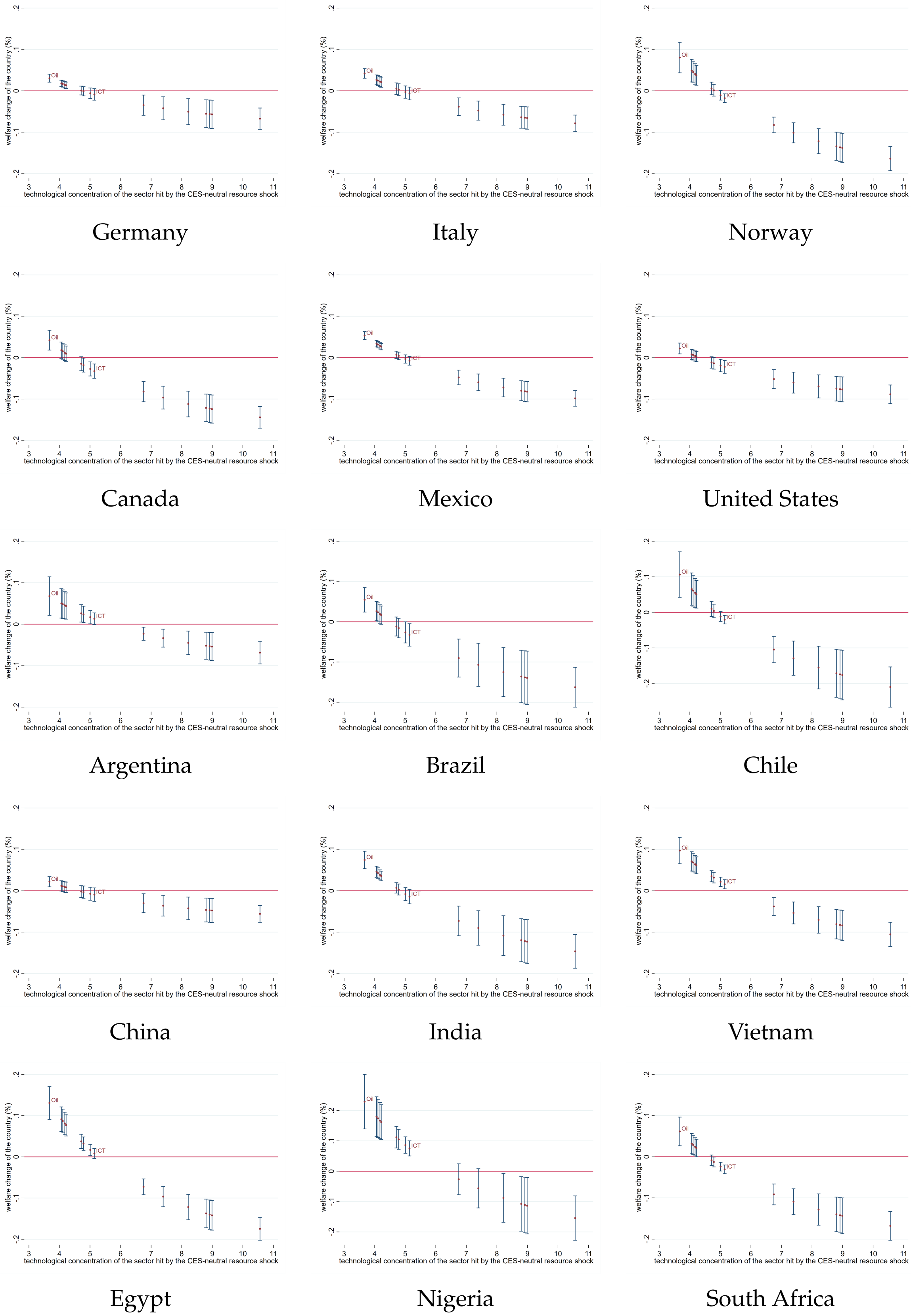


Figure 6: Welfare responses to restrictive resource shocks, in selected economies

sumption that firms' labor unit input requirements are (inverse) Pareto distributed, it leads to a simple expression of national welfare as a function of a very limited number of sufficient statistics. The Pareto assumption also allows for measuring the concentration of such technological coefficients across firms through a single exogenous parameter.

Using "CES-neutral"to refer to a resource shock that does not change welfare under CES, we can summarize our results as follows. If an expansionary CES-neutral domestic resource shock hits a sector with low technological concentration, a country may still experience immiserizing growth, that is, a welfare loss under IPT. Vice versa, if a contractionary CES-neutral domestic resource shock hits a sector with low technological concentration, the country may still experience enriching decline, that is, a welfare gain under IPT. These results are derived both theoretically and empirically for resource shocks of realistic magnitude as proof of concept.

Despite these novel insights, in this paper we have not exploited the full potential of our model, which is ready for full-fledged positive and normative quantitative exercises based on calibration, validation and simulation of all kinds of counterfactual scenarios, without much additional complexity with respect to the commonly used CES-based quantitative trade models. We leave these exercises to future exploration.

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Appendices

A Derivations

In this section we describe in detail the derivations of the expression for expenditure shares and welfare.

A.1 Sectoral expenditure share

Given utility (1) and budget constraint (2), a consumer of country l allocates expenditure over a quantity $q^c_{jl}(i;z) \geq 0$ based on the following first order conditions:

$$\frac{\beta(z)U_l}{\sum_{j=1}^{J} u_{jl}(z)} \left(\alpha - \gamma q^c_{jl}(i;z)\right) = \lambda_l p_{jl}(i;z) \ , \ \forall (j,z),$$

$$\sum_{z=1}^{Z} \sum_{j=1}^{J} \int_0^{N_{jl}(z)} p_{jl}(i;z)\, q^c_{jl}(i;z)\, di = w_l \ ,$$

given a binding budget constraint, i.e. the marginal utility of income is $\lambda_l > 0$. Introducing the definitions of sectoral quantity index $\mathbb{Q}_l(z)$ and sectoral price index $\mathbb{P}_l(z)$ yields:

$$\frac{\beta(z)U_l}{\mathbb{Q}_l(z)} \left(\alpha - \gamma q^c_{jl}(i;z)\right) = \lambda_l p_{jl}(i;z) \ , \ \forall (j,z),$$

$$\sum_{z=1}^{Z} \mathbb{P}_l(z)\mathbb{Q}_l(z) = w_l \ .$$

The choke price $\hat{p}_l(z) > 0$ is defined as the minimum (finite) price at which the consumer of country l optimally allocates zero consumption on a variety of sector z. Thus, evaluating the first order condition for consumption at the choke price for any pair of sectors, z' and z'', and then taking the ratio yields:

$$\frac{\hat{p}_l(z'')}{\hat{p}_l(z')} \frac{\mathbb{Q}_l(z'')}{\mathbb{Q}_l(z')} = \frac{\beta(z'')}{\beta(z')}.$$

The definition $\eta_l(z) \equiv \mathbb{P}_l(z)/\hat{p}_l(z)$ implies:

$$\frac{\mathbb{P}_l(z'')\mathbb{Q}_l(z'')}{\mathbb{P}_l(z')\mathbb{Q}_l(z')} = \frac{\beta(z'')\eta(z'')}{\beta(z')\eta(z')}.$$

Substituting in the budget constraint yields:

$$\sum_{z''=1}^{Z} \mathbb{P}_l(z'')\mathbb{Q}_l(z'') = \frac{\mathbb{P}_l(z')\mathbb{Q}_l(z')}{\beta(z')\eta(z')} \sum_{z''=1}^{Z} \beta(z'')\eta(z'') = w_l \ ,$$

$$\mathbb{P}_l(z')\mathbb{Q}_l(z') = \frac{\beta(z')\eta(z')}{\sum_{z''=1}^{Z} \beta(z'')\eta(z'')} w_l.$$

Therefore, the expenditure share in goods from any sector z' is given by $\theta(z') = \frac{\beta(z')\eta(z')}{\sum_{z''=1}^{Z}\beta(z'')\eta(z'')}$.

A.2 Aggregate variables given an Inverse Pareto distribution

Expenditure and utility due to individual consumption in country l on goods from sector z sourced from country j are:

$$\begin{aligned} e_{jl}(z) &= \frac{\alpha}{\gamma}\left(\mu_1(z) - \mu_2(z)\right)\hat{p}_l(z)N_{jl}(z) \\ u_{jl}(z) &= \frac{\alpha^2}{2\gamma}\left(1 - \mu_2(z)\right)N_{jl}(z). \end{aligned}$$

Quantity index and price index in a certain country l and sector z are

$$\begin{aligned} \mathbb{Q}_l(z) &\equiv (1/\alpha)\sum_{j=1}^{J} u_{jl}(z) = \frac{\alpha}{2\gamma}\left(1 - \mu_2(z)\right)N_l(z), \\ \mathbb{P}_l(z) &\equiv \sum_{j=1}^{J} e_{jl}(z)/\mathbb{Q}_l(z) = \left(\frac{2(\mu_1(z) - \mu_2(z))}{1 - \mu_2(z)}\right)\hat{p}_l(z). \end{aligned}$$

The system of output market clearing $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta(z)w_l$ and choke price $\hat{p}_l(z) = w_l c_l^*(z)$ yields the measure of varieties of sector z available in country l sourced from anywhere:

$$N_l(z) = \frac{\gamma}{\alpha}\frac{\theta(z)}{\left(\mu_1(z) - \mu_2(z)\right)c_l^*(z)}\ .$$

Aggregate revenue made by firms producing in country j sector z and selling to country l is given by $R_{jl}(z) \equiv N_{jl}(z)\int_0^{c_{jl}^*(z)} r_{jl}(c;z)dG_j(c;z)/G_j(c_{jl}^*(z);z)$ and the corresponding aggregate profit is given by $\Pi_{jl}(z) \equiv N_{jl}(z)\int_0^{c_{jl}^*(z)} \pi_{jl}(c;z)dG_j(c;z)/G_j(c_{jl}^*(z);z)$,

$$\begin{aligned} R_{jl}(z) &= \underbrace{N_j^E(z)[\tau_{jl}(z)w_j c_j^{max}(z)]^{-k(z)}\hat{p}_l(z)^{k(z)}}_{\text{mass of exporters}} \times \underbrace{\zeta_X(z)\hat{p}_l(z)L_l}_{\text{average revenue}} \\ \Pi_{jl}(z) &= \underbrace{N_j^E(z)[\tau_{jl}(z)w_j c_j^{max}(z)]^{-k(z)}\hat{p}_l(z)^{k(z)}}_{\text{mass of exporters}} \times \underbrace{\zeta_\Pi(z)\hat{p}_l(z)L_l}_{\text{average profit}} \end{aligned}$$

with $\zeta_X(z) \equiv k(z)\left(\frac{1}{k(z)} - \frac{1}{k(z)+2}\right)\frac{\alpha}{4\gamma}$ and $\zeta_\Pi(z) \equiv k(z)\left(\frac{1}{k(z)} - \frac{2}{k(z)+1} + \frac{1}{k(z)+2}\right)\frac{\alpha}{4\gamma}$. To obtain these expressions, we have substituted for the fraction of entrants in country j that become exporters to country l, $N_{jl}(z) = (c_{jl}^*(z)/c_j^{max}(z))^{k(z)}N_j^E(z)$, and then for the corresponding export cutoff $c_{jl}^*(z) = \frac{\hat{p}_l(z)}{\tau_{jl}(z)w_j}$. Moreover, we have followed equation (20) in Melitz and Redding (2013) to decompose extensive and intensive margins.

The implication is that aggregate profits are a constant fraction $\Pi_{jl}(z) = \delta(z)R_{jl}(z)$ of aggregate revenue, as $\delta(z) \equiv \zeta_\Pi(z)/\zeta_X(z)$ is fixed by the exogenous concentration parameter of the technological distribution and does not vary by country. Not only $\zeta_X(z)$ and $\zeta_\Pi(z)$ but also $\delta(z)$ are decreasing functions of $k(z)$.

A.3 Equilibrium with diversification

An equilibrium with diversification is characterized by a strictly positive entry of firms in every country and sector pair, such that $N_j^E(z) > 0$ for all $j = 1, ..., J$ and $z = 1, ..., Z$.

Autarky. The special case of countries that do not trade is an equilibrium with diversification, since in every country there is positive demand for every sector and this cannot be satisfied by imports by definition. Therefore, there exists a set of trade costs $\{\tau_{jl}(z) \geq 1 : j, l = 1, ..., J,\ z = 1, ..., Z\}$ for which an equilibrium with diversification exists.

Moreover, this equilibrium is unique. Since countries are disconnected, there is no relationship between their nominal wages, therefore, the wage in every country can arbitrarily be set to 1, as the numeraire in its own market. This can be seen by replacing prohibitive trade costs in the equilibrium conditions FEC* and OMC*, then noticing that any nominal wage drops from the equations. The solution is:

$$w_j = 1\ \forall j\,,\ c_j^{aut}(z) = \left(\frac{c_j^{max}(z)^{k(z)}}{\zeta_\Pi(z) L_j / f_j} \right)^{\frac{1}{1+k(z)}}\ ,\ N_j^{E\ aut}(z) = \theta(z)\delta(z)\frac{L_j}{f_j}\ \forall (j, z).$$

Uniqueness in open economy. Assume that an open economy equilibrium with diversification exits and there are two different vectors of relative wages, $\boldsymbol{a} = (1, a_2, ..., a_J)$ and $\boldsymbol{b} = (1, b_2, ..., b_J)$, that are both an equilibrium. This means that the labor market clearing conditions must hold in each country. Let $f(\cdot; z) : \Re_+^{J-1} \to \Re_+$ be the continuous function describing the relative labor demand (left hand side of LMC**) after substituting for the measures of entrants $(y_1(z), y_2(z), ..., y_J(z))$ implied by FEC** and OMC**. Highlight with superscript $^{(+)}$ or $^{(-)}$ the sign of functional dependence on the corresponding relative wage.
Consider first the labor market equilibrium conditions with $J = 2$ countries:

$$\text{LMC 1:}\ \sum_{z=1}^{Z} f_1\left(1, a_2^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_1\left(1, b_2^{(+)}; z\right) = 1$$
$$\text{LMC 2:}\ \sum_{z=1}^{Z} f_2\left(1, a_2^{(-)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_2\left(1, b_2^{(-)}; z\right) = 1$$

If $\boldsymbol{a}$ is an equilibrium and $b_2 < a_2$ applies, then LMC in country 1 does not hold (demand is too low) and LMC in country 2 also fails (demand is too high).
Consider now the labor market equilibrium with $J = 3$ countries:

$$\text{LMC 1:}\ \sum_{z=1}^{Z} f_1\left(1, a_2^{(+)}, a_3^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_1\left(1, b_2^{(+)}, b_3^{(+)}; z\right) = 1$$
$$\text{LMC 2:}\ \sum_{z=1}^{Z} f_2\left(1, a_2^{(-)}, a_3^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_2\left(1, b_2^{(-)}, b_3^{(+)}; z\right) = 1$$
$$\text{LMC 3:}\ \sum_{z=1}^{Z} f_3\left(1, a_2^{(+)}, a_3^{(-)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_3\left(1, b_2^{(+)}, b_3^{(-)}; z\right) = 1.$$

If $\boldsymbol{a}$ is an equilibrium and $b_2 < a_2$ applies, then $b_3 > a_3$ is necessary, otherwise LMC in country 1 does not hold. However, if $b_2 < a_2$ and $b_3 > a_3$ apply, labor demand in country 2 is too high while labor demand in country 3 is too small.
Consider finally the labor market equilibrium with $J = 4$ countries:

$$\begin{aligned}
&\text{LMC 1:} \quad \sum_{z=1}^{Z} f_1\left(1, a_2^{(+)}, a_3^{(+)}, a_4^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_1\left(1, b_2^{(+)}, b_3^{(+)}, b_4^{(+)}; z\right) = 1 \\
&\text{LMC 2:} \quad \sum_{z=1}^{Z} f_2\left(1, a_2^{(-)}, a_3^{(+)}, a_4^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_2\left(1, b_2^{(-)}, b_3^{(+)}, b_4^{(+)}; z\right) = 1 \\
&\text{LMC 3:} \quad \sum_{z=1}^{Z} f_3\left(1, a_2^{(+)}, a_3^{(-)}, a_4^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_3\left(1, b_2^{(+)}, b_3^{(-)}, b_4^{(+)}; z\right) = 1. \\
&\text{LMC 4:} \quad \sum_{z=1}^{Z} f_4\left(1, a_2^{(+)}, a_3^{(+)}, a_4^{(-)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_4\left(1, b_2^{(+)}, b_3^{(+)}, b_4^{(-)}; z\right) = 1.
\end{aligned}$$

If $\boldsymbol{a}$ is an equilibrium and $b_2 < a_2$ applies, then at least $b_3 > a_3$ or $b_4 > a_4$ is necessary, otherwise LMC in country 1 does not hold. However, in both cases, labor demand in country 2 is too high, if no other change - of the opposite sign - occurs. Therefore, either $b_3 > a_3$ and $b_4 < a_4$ apply or $b_3 < a_3$ and $b_4 > a_4$ apply. If $b_2 < a_2$, $b_3 > a_3$ and $b_4 < a_4$ apply, then labor demand in country 3 is too small. Otherwise, if $b_2 < a_2$, $b_3 < a_3$ and $b_4 > a_4$ apply, then labor demand in country 4 is too small.

By induction, we conclude that, regardless of the number of countries, if $\boldsymbol{a}$ is an equilibrium, then any other vector $\boldsymbol{b} \neq \boldsymbol{a}$ is not an equilibrium since the labor market in at least one country does not clear.

Existence in open economy. An open economy equilibrium with diversification exists only if, for all (m, z), we have:

$$x_m(z) > 0 \iff \frac{w_m}{w_1} > \left[\sum_{l \neq m}^{J} \frac{K_{ml}(z)}{T_{ml}(z)} \left(\frac{w_l}{w_1}\right)^{1+k(z)} x_l(z)\right]^{\frac{1}{1+k(z)}} > 0 \tag{43}$$

$$y_m(z) > 0 \iff \frac{w_m}{w_1} < \left[x_m(z) \sum_{j \neq m}^{J} \frac{K_{jm}(z) E_{jm}(z)}{T_{jm}(z)} \left(\frac{w_1}{w_j}\right)^{k(z)} y_j(z)\right]^{-\frac{1}{k(z)}}. \tag{44}$$

Since existence of an equilibrium postulates that FEC** and OMC** are satisfied, then the previous conditions are equivalent to:

$$x_m(z) > 0 \iff \frac{w_m}{w_1} > [1 - x_m(z)]^{\frac{1}{1+k(z)}} \qquad \forall (m, z) \tag{45}$$

$$y_m(z) > 0 \iff \frac{w_m}{w_1} < [1 - x_m(z) y_m(z)]^{-\frac{1}{k(z)}} \quad \forall (m, z) \tag{46}$$

which are the expressions (26) and (27).

A special case. Consider the special case in which countries pay the same wage, i.e. $w_j = w_1 = 1$ for all j, and there is no sector in which a country experiences more entry in open economy relative to autarky, therefore $y_j(z) = 1$ for all (j, z). The necessary and sufficient condition (28) is satisfied for $0 < x_j(z) < 1$ for all countries and sectors (j, z).

In this case, if an equilibrium exists, then, by definition, wages are the same across countries. Furthermore, condition (24) implies that it is an equilibrium with diversification $0 < x_j(z) < 1$ for all countries and sectors (j, z). The system of necessary conditions (26) and (27) simplifies to:

$$\sum_{l \neq m}^{J} \frac{K_{ml}(z)}{T_{ml}(z)} < \frac{w_m}{w_1} = 1 < \left[\sum_{j \neq m}^{J} \frac{K_{jm}(z) E_{jm}(z)}{T_{jm}(z)} \right]^{-1} \qquad \forall (m, z),$$

which can be assessed based on the model's parameters only. Clearly as technological differences attenuate and trade costs increase while remaining finite (i.e. $K_{ml}(z)/T_{ml}(z) \to 0_+$ and $K_{jm}(z)/T_{jm}(z) \to 0_+$ for all z), the feasible support for each relative wage widens.

A further special case is one in which countries are symmetric in their characteristics and face a common bilateral trade cost $\tau > 1$. Due to symmetry, the system of necessary conditions (26) and (27) simplifies to:

$$\frac{J-1}{\tau} < \frac{w_m}{w_1} = 1 < \frac{\tau}{J-1} \qquad \forall m.$$

Therefore, $\tau > J - 1$ is the finite (hence not prohibitive) level of trade cost such that the sufficient condition for existence of an open economy equilibrium with diversification is satisfied. Note that, for the classical example with $J = 2$ countries, the presence of any trade cost $\tau > 1$ is sufficient.

Outside of diversification. Assume that, given the fundamentals of the economy, there are some pairs of country and sector that do not feature production. Given free entry and a continuous distribution of cutoff costs, this means that there is no positive measure of entrants for a feasible cost cutoff in at least one country-sector pair, i.e. $N_j^E(z) = 0$ for every $c_j^*(z) > 0$.

Since having no entrants means that there are no incumbent firms, country-sector pairs with no production simply do not generate income. Yet, the labor market clearing condition must hold, irrespective of the fact that the equilibrium features diversification or not. The system (12)-(14) "fails" to characterize an equilibrium without diversification because the free entry condition and the output market clearing condition are not well-posed.

More precisely, the free entry condition is misspecified because it postulates the existence of a full matrix of strictly positive country-sector cutoff costs, but this is true if and only if there is a positive mass of entrants. The output market clearing condition fails because it is "coupled" with the free entry condition through - again - a postulated matrix of cutoff costs. Therefore, it is not enough to simply replace $N_j^E(z) = 0$ in the output market clearing condition. As to this condition should correspond a missing cost cutoff $c_j^*(z)$, the free entry condition for country j's sector z should be removed from the system (12)-(14), and this implies solving a different problem than the original one with a zero measure of entrants for the pair (j, z).

To solve the equilibrium of the model without assuming diversification, one needs to assess when entry fails "before" writing the system of free entry conditions. For this assessment, note that free entry with a continuous distribution of costs over the positive support (so that $c^*_j(z)$ can be arbitrarily close to zero) implies that the output of every country-sector pair is sold to every country. Consequently, the matrix of trade flows must also be full. If one knows which trade flows should be zero (both in the observation and in the model's predictions), then one also knows which free entry conditions to remove and which measure of entrants to set to zero. The resulting "truncated" version of (12)-(14) is still characterized by as many equations as unknowns.

An interesting restriction. Any pair of sub-utility bundles $u_{jl}(z)$ and $u_{kl}(z)$ are perfect substitutes in the utility (1) of consumers in country l. The prices of these sub-utility bundles are, respectively, $e_{jl}(z)/u_{jl}(z) = \frac{\alpha}{2}\frac{\bar{p}_{jl}(z)-\bar{\bar{p}}_{jl}(z)}{1-\bar{\bar{p}}_{jl}(z)}\hat{p}_l(z)$ and $e_{kl}(z)/u_{kl}(z) = \frac{\alpha}{2}\frac{\bar{p}_{kl}(z)-\bar{\bar{p}}_{kl}(z)}{1-\bar{\bar{p}}_{kl}(z)}\hat{p}_l(z)$. Therefore, $\frac{\bar{p}_{jl}(z)-\bar{\bar{p}}_{jl}(z)}{1-\bar{\bar{p}}_{jl}(z)} = \frac{\bar{p}_{kl}(z)-\bar{\bar{p}}_{kl}(z)}{1-\bar{\bar{p}}_{kl}(z)}$ is a necessary condition for country l to source goods from sector z from both origins j and k, which must be true if the matrix of trade flows should be full. Interestingly, the assumption of an Inverse Pareto distribution of technology implies that this restriction is satisfied mechanically, since moments of the relative price distribution only depend on the sector.

Bilateral trade balance. $\sum_{z=1}^{Z} R_{jl}(z)$ equals total import of country l from country j. Then, total import of country l from country j, and total export of country l to country j are equal if and only if the bilateral trade balance condition holds:

$$\text{BTB}: \quad \sum_{z=1}^{Z} R_{jl}(z) = \sum_{z=1}^{Z} R_{lj}(z) \quad \forall (j,l). \tag{47}$$

Two remarks are important. First, output market clearing in each country and sector pair implies that the country-level budget constraint is satisfied. To see this, write a sectoral output market clearing condition for a country j

$$\sum_{m=1}^{J} N_{mj}(z) \int_0^{c^*_{mj}(z)} r_{mj}(c;z) \frac{dG_m(c;z)}{G_m(c^*_{mj}(z);z)} = \theta_j(z) w_j L_j \,,$$

and sum over sectors:

$$\underbrace{\sum_{z=1}^{Z}\sum_{m=1}^{J} N_{mj}(z) \int_0^{c^*_{mj}(z)} r_{mj}(c;z) \frac{dG_m(c;z)}{G_m(c^*_{mj}(z);z)}}_{\text{total expenditure of country } j} = w_j L_j \,.$$

Second, output market clearing, free entry and bilateral trade balance imply labor market clearing at the country level. To see this, write a sectoral output market clearing condition for a country j, sum over sectors and recognize the expression for total imports from a certain origin:

$$\sum_{m=1}^{J} \underbrace{\left(\sum_{z=1}^{Z} N_{mj}(z) \int_0^{c^*_{mj}(z)} r_{mj}(c;z) \frac{dG_m(c;z)}{G_m(c^*_{mj}(z);z)} \right)}_{\text{total import of country } j \text{ from country } m} = w_j L_j \,.$$

If there is trade balance between countries, then total import of country j from country m must be equal to total export of country j to country m

$$\sum_{m=1}^{J} \underbrace{\left(\sum_{z=1}^{Z} N_{jm}(z) \int_0^{c^*_{jm}(z)} r_{jm}(c;z) \frac{dG_j(c;z)}{G_j(c^*_{jm}(z);z)} \right)}_{\text{total export of country } j \text{ to country } m} = w_j L_j \; .$$

Thus, substituting for the measure of exporters $N_{jm}(z) = G_j(c^*_{jm}(z);z) N_j^E(z)$, yields the labor market clearing condition in country j:

$$\sum_{z=1}^{Z} N_j^E(z) \left(\sum_{l=1}^{J} \int_0^{c^*_{jl}(z)} r_{jl}(c;z) dG_j(c;z) \right) = w_j L_j \; .$$

Hence, in autarky the bilateral trade balance condition is redundant by definition. In open economy with aggregate trade balance, either the bilateral trade balance condition or the labor market clearing condition is redundant because it is implied by the other equilibrium conditions.

A.4 Comparative statics of a resource shock holding the vector of wages constant

Given the following representation of FEC**

$$\sum_{l=1}^{J} a^0_{hl}(s) \left(\frac{x^1_l(s)}{x^0_l(s)} \right) = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)} \right)^{k(s)} < 1,$$

$$\text{where } a^0_{hl}(s) \equiv \frac{K^0_{hl}(s)}{T^0_{hl}(s)} \left(\frac{w^0_l}{w^0_h} \right)^{1+k(s)} x^0_l(s) \text{ and } \sum_{l=1}^{J} a^0_{hl}(s) = 1,$$

define $A^0(s)$ to be the J-dimensional matrix with row-j and column-l element $a^0_{jl}(s)$, whose entries sum to one on each row. Call $A^0_l(s)$ the matrix constructed from $A^0(s)$ by replacing the l-th column with a vector of J entries all equal to 1 and call $A^1_l(s)$ the matrix constructed from $A^0(s)$ by replacing the l-th column with a vector of J entries all equal to $\left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)} < 1$. Define the column vectors $\boldsymbol{z}^i(s) = \{z^i_l(s) = x^i_l(s)/x^0_l(s) : l = 1, ..., J\}$ and $\boldsymbol{c}^i(s) = \{c^i_l(s) = (c_h^{max\ i}(s)/c_h^{max\ 0}(s))^{k(s)} : l = 1, ..., J\}$, for the two regimes $i = \{0, 1\}$ before and after the shock, then the FEC** takes the form of a linear system:

$$A^0(s)\boldsymbol{z}^i(s) = \boldsymbol{c}^i(s)$$

whose solution is obtained by Cramer's rule

$$z^i_l(s) = \frac{det A^i_l(s)}{det A^0(s)}$$

and note that $det A^0_l(s) = det A^0(s)$ by construction. Call $TA^0_l(s)$ the transpose of $A^0_l(s)$ and $TA^1_l(s)$ the transpose of $A^1_l(s)$. Since $TA^1_l(s)$ is the matrix which results from multiplying one row of $TA_l^0(s)$ by the scalar $\left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)}$ then the determinant satisfies $detTA_l^1(s) = \left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)} detTA_l^0(s)$. Since the determinant of the transpose is equal to the determinant of the original matrix then $detA_l^1(s) = \left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)} detA_l^0(s)$. This shows that:

$$z_l^1(s) = \frac{x_l^1(s)}{x_l^0(s)} = \frac{detA_l^1(s)}{detA^0(s)} = \frac{detTA_l^1(s)}{detTA_l^0(s)} = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\right)^{k(s)} < 1\ \forall l.$$

Substituting for the definition of $x_l(s)$

$$\frac{x_l^1(s)}{x_l^0(s)} = \left(\frac{c_l^{*1}(s)}{c_l^{*0}(s)}\right)\left(\frac{c_l^{max\ 0}(s)}{c_l^{max\ 1}(s)}\right)^{k(s)}$$

yields (34).

Given the following representation of the OMC**

$$\sum_{j=1}^{J} b_{jh}^0(s)\frac{y_j^1(s)}{y_j^0(s)} = \frac{x_h^0(s)}{x_h^1(s)}\left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{k(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1$$

where $b_{jh}^0(s) \equiv \dfrac{K_{jh}^0(s)E_{jh}^0(s)}{T_{jh}^0(s)}\left(\dfrac{w_h^0}{w_j^0}\right)^{k(s)} x_h^0(s)y_j^0(s)$ and $\sum_{j=1}^{J} b_{jh}^0(s) = 1$,

define $TB^0(s)$ the J-dimensional matrix with row-l and column-j element $b_{jl}^0(s)$, whose entries sum to one on each row. Call $TB_j^0(s)$ the matrix constructed from $TB^0(s)$ by replacing the j-th column with a vector of J entries all equal to 1 and call $TB_j^1(s)$ the matrix constructed from $TB^0(s)$ by replacing the j-th column with a vector of J entries all equal to $\left(c_h^{max\ 0}(s)/c_h^{max\ 1}(s)\right)^{2k(s)} > 1$. Define the column vectors $\boldsymbol{t}^i(s) = \{t_j^i(s) = y_j^i(s)/y_j^0(s) : j = 1, ..., J\}$ and $\boldsymbol{d}^i(s) = \{d_j^i(s) = (c_h^{max\ 0}(s)/c_h^{max\ i}(s))^{2k(s)} : j = 1, ..., J\}$, for the two regimes $i = \{0, 1\}$ before and after the shock, then the OMC** takes the form of the linear system

$$TB^0(s)\boldsymbol{t}^i(s) = \boldsymbol{d}^i(s).$$

Cramer's rule yields the solution

$$t_j^i(s) = \frac{detTB_j^i(s)}{detTB^0(s)}$$

and note that $detTB_j^0(s) = detTB^0(s)$ by construction. Since the determinant of the transpose is equal to the determinant of the original matrix then $detTB_j^0(s) = detB_j^0(s)$ and $detTB_j^1(s) = detB_j^1(s)$, where $B_j^i(s)$ is the transpose of $TB_j^i(s)$. Note that $B_j^1(s)$ is the matrix which results from multiplying one row of $B_j^0(s)$ by the scalar $(c_h^{max\ 0}(s)/c_h^{max\ i}(s))^{2k(s)}$ then the determinant satisfies $detB_j^1(s) = (c_h^{max\ 0}(s)/c_h^{max\ i}(s))^{2k(s)} detTB_j^0(s)$. This shows that:

$$\frac{y_j^1(s)}{y_j^0(s)} = \frac{detTB_j^1(s)}{detTB^0(s)} = \frac{detB_j^1(s)}{detB_j^0(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1\ \forall j.$$

Substituting for the definition of $y_j(s)$ yields (35).

A.5 One sector open economy

In a one-sector economy, a fixed labor supply trivially fixes labor demand in the one sector. This simplifies OMC** which is now a condition on relative wage and cutoff costs only as FEC**:

$$\begin{aligned}
\text{FEC**} \quad &: \quad \sum_{l=1}^{J} \frac{K_{jl}}{T_{jl}} \left(\frac{w_l}{w_j}\right)^{1+k} x_l = 1 \qquad && \forall j, \\
\text{OMC**} \quad &: \quad x_l \sum_{j=1}^{J} \frac{K_{jl}E_{jl}}{T_{jl}} \left(\frac{w_l}{w_j}\right)^{k} = 1 \qquad && \forall l, \\
\text{LMC**} \quad &: \quad y_j = 1 \qquad && \forall j.
\end{aligned}$$

The system reduces to

$$w_j L_j = \sum_{l=1}^{J} \left(\frac{\frac{K_{jl}}{\tau_{jl}^k} \left(\frac{1}{w_j}\right)^k}{\sum_{j=1}^{J} \frac{K_{jl}}{\tau_{jl}^k} \left(\frac{1}{w_j}\right)^k} \right) w_l L_l \quad \forall j.$$

Using labor in country 1 as the numeraire, such that $w_1 = 1$ yields

$$L_1 = \sum_{l=1}^{J} \left(\frac{\frac{K_{1l}}{\tau_{1l}^k}}{\sum_{j=1}^{J} \frac{K_{jl}}{\tau_{jl}^k} \left(\frac{1}{w_j}\right)^k} \right) w_l L_l.$$

Note that $K_{jl}/K_{1l} = K_{j1}$ does not depend on l, and a decomposition of trade costs $\tau_{jl} = \tau_j \tau_l$ implies that $\tau_{1l}/\tau_{jl} = \tau_1/\tau_j$, also does not depend on l. Under this parametrization, the wage can be obtained in closed form:

$$w_j = \left(\frac{\tau_1^k K_{j1}}{\tau_j^k} \frac{L_1}{L_j}\right)^{\frac{1}{1+k}} = \left(\left(\frac{\tau_1 c_1^{max}}{\tau_j c_j^{max}}\right)^k \frac{f_1}{f_j}\right)^{\frac{1}{1+k}} \quad \forall j,$$

and it is a decreasing function of the upper bound of the cost support c_j^{max}, of fixed cost f_j and of the trade cost τ_j.

B Relationship to ACR (2012) and ACDR (2018)

In this section we clarify the relationship between our model and the class of models discussed in Arkolakis et al. (2012) and extended in Arkolakis et al. (2019).

B.1 The class of models considered in ACDR (2012)

It is immediate to conclude that the so-called macro restrictions in their setup hold in our framework: total value of imports is equal to total value of exports (R1); in each sector aggregate profits are a constant fraction of aggregate revenue (R2); the gravity equation implied by the model, once written in terms of the measure of potential entrants, has a canonical structure (R3′ that is the stronger form of R3). In particular, the latter applies thanks to the adoption of a Pareto distribution of technology that makes the moments of the relative price distribution depend only on the concentration parameter. In our model, as in their analysis, (i) the cost function at the firm level is linear, (ii) labor is the only factor of production, (iii) the labor market is competitive, while (vi) the output market has a monopolistically competitive structure.

Indeed, there is only one primitive of the theory in which our model deviates from the class of models considered in Arkolakis et al. (2012): preferences across varieties are represented by an additive-separable utility function that features a variable elasticity of substitution.

B.2 The class of models considered in ACDR (2018)

The existence of a finite choke price and the adoption of the Pareto distribution place the model within the class of those discussed in Arkolakis et al. (2019). To show this, in what follows we rewrite the salient feature of our framework using their approach.

With reference to the group of firms producing in country j sector z and selling to a destination l, call $v \equiv \hat{p}_l(z)/[\tau_{jl}(z)w_j c] = c^*_{jl}(z)/c \geq 1$ the measure of efficiency of a firm endowed with productivity $1/c$ relative to the other firms in the group. Call $\mu_{jl}(v;z) = p_{jl}(c;z)/[\tau_{jl}(z)w_j c]$ the function describing the markup factor as a function of relative efficiency. After substituting for $c = c^*_{jl}(z)/v$, the markup factor looses its dependence on origin, destination or sector

$$\mu(v) = \frac{1}{2}\left(1 + \frac{c^*_{jl}(z)}{c}\right) = \frac{1+v}{2},$$

as the three channels of dependence are captured by the cutoff cost, and the same holds true for the elasticity of the markup factor with respect to relative efficiency:

$$\frac{d\ln\mu(v)}{d\ln(v)} = \frac{v}{1+v}.$$

The individual Marshallian demand function is described by a demand shifter $Q \equiv \frac{\alpha}{\gamma}$ and a decreasing function $D\left(\mu(v)/v\right) \equiv 1 - \mu(v)/v = (v-1)/(2v)$ such that total sales and profits are given by:

$$r_{jl}\left(c,v;z\right) = L_l Q \tau_{jl}\left(z\right) w_j c\mu(v) D\left(\mu(v)/v\right)$$
$$\pi_{jl}\left(c,v;z\right) = [(\mu(v)-1)/\mu(v)] r_{jl}\left(c;z\right).$$

B.3 A foreign trade shock

The implications of incomplete pass-through for welfare following a foreign trade shock can be illustrated by replicating the "ex-ante" conjecture in Arkolakis et al. (2012), that consists of the limit counterfactual exercise of "*moving to autarky*" given the same fixed entry cost, i.e. f_j, and labor endowment, i.e. L_j, and support of the technological distribution $c_j^{max}(z)$. By definition $\lambda_{jj}^{aut}(z) \equiv 1$ while the allocation of labor across sectors is proportional to the measure of firms $\rho_j^{aut}(z) = f_j N_j^{E\ aut}(z)/[\delta(z)L_j] = \theta(z)$. Thus, welfare in autarky is given by

$$W_j^{aut} = \prod_{z=1}^{Z} B(z) \left(\frac{f_j c_j^{max}(z)^{k(z)}}{L_j} \frac{1}{\theta(z)} \right)^{-\frac{\beta(z)}{k(z)} \frac{k(z)}{1+k(z)}}$$

and the measured welfare cost for country j of a shock that all-and-only shuts down trade linkages is given by:

$$\frac{W_j^{aut}}{W_j} = \prod_{z=1}^{Z} \left(\frac{\lambda_{jj}(z)}{\rho_j(z)/\theta(z)} \right)^{\frac{\beta(z)}{k(z)} \frac{k(z)}{1+k(z)}}. \tag{48}$$

Note that expenditure shares matter for an accounting of the cost of moving to autarky. Expenditure shares, e.g. $\theta(z)$, are lower than the corresponding Cobb-Douglas shares, e.g. $\beta(z)$, for sector with lower technological concentration, hence with lower pass-through. Therefore, sectors characterized by lower technological concentration attenuate the measurement of autarky-induced welfare changes, both due to lower pass-true and a comparatively smaller expenditure share.

These differential effects would be absent if technological concentration was the same across sectors, and the closed results provided in Arkolakis et al. (2019) are confined to that scenario. Thus, the tractability of the present framework sheds light on the role played by heterogeneity of technological concentration across sectors on the measurement of autarky-induced welfare changes.