Trade and the Labor Market: A Dynamic Model with on-the-job Search

Davide Suverato *†

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Abstract

This paper develops a dynamic general equilibrium model of trade with heterogeneous firms and homogeneous workers who search for a job also when they are employed; *on the job search* (OJS). The model is able to predict the short–run costs due to labor market adjustment and the long–run gains from increased trade exposure.

The model shows how the destruction of jobs, caused by trade–induced firm exit, determines a rise in unemployment and a reduction in the probability to find a job. As a result, welfare decreases in the short–run. However, this allocation is unstable. The excessively low labor market tightness triggers the recovery. During the adjustment, the probability of receiving a wage offer grows, the wages increase and new exporters expand employment also at the expenses of relatively low productivity firms which shrink. In the long–run, trade increases welfare; not only because of a greater aggregate productivity, but also because the initial loss of jobs is offset and the average wage is higher.

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*LMU University of Munich, e-mail address: davide.suverato@econ.lmu.de

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1 Introduction

Recent empirical research using matched employer–employee data has documented how the response of the labor market to international trade follows a long–lasting dynamics, with the potential of hurting welfare in the short–run. In a study on the Brazilian trade liberalization, Dix-Carneiro (2014) estimates a trade–induced welfare increase of about 2.4% in the long–run. Nevertheless, at the impact of increased trade exposure manufacturing workers experience a loss of welfare up to 9.9%, and the cost from the adjustment of the labor market eats up to 26% of the total trade–induced welfare creation. Short–run costs emerge also when a developed and already open economy becomes more internationally integrated. Autor et al. (2013), Autor et al. (2014) and Acemoglu et al. (2016), find that the increased trade exposure to China induced a massive loss of US manufacturing jobs (about 18% of sectoral employment) and a reduction of 2.14% in annual earnings. This evidence does not contradict the view of long–run welfare gains from trade. It rather sheds light on the short–run loss of jobs and on the wage cut which originate from the same root of these gains.

What Trefler (2004) called the conflict between the short–run adjustment costs and the long–run efficiency gains from trade is emerging with all its strength nowadays. The fear of international trade pervades the working class in many countries (and it dominates the recent political agenda in many countries). Although this might be tough to admit, this should not sound surprising since gains from trade come at the cost of labor market displacement. A deeper concern is instead that classical theories which identify winners and losers from trade based on sectoral and skill specificity do not offer much guidance through the current challenges. In fact they are less helpful precisely where most of the labor market adjustment to trade takes place, that is within–sector, see Levinsohn (1999), Haltiwanger et al. (2004) and Wacziarg and Wallack (2004), and across–workers with similar observable characteristics, as documented by Helpman et al. (2012).

Following this motivation, trade models have been recently extended to account for labor market frictions, which yields a better understanding of the impact of trade on unemployment. However, these studies are confined to a steady state analysis, thus they do not explain the short–run costs of labor market adjustment. In contrast, the response of the labor market to increased trade exposure has arguably a transition phase of a decade at least.[1] It is during this period that welfare costs hit, and it is a too long span of a working life to be overlooked.

[1]Trefler (2004) studies the US–Canada free trade agreement and finds substantial job losses in the short–run (up to 12% of the pre–reform sectoral employment) which the economy compensates for only about a decade after the agreement. The same study reports a 15% increase of labor productivity and a 3% increase in wages after ten years. Dix-Carneiro (2014) shows that the aggregate welfare covers most of the convergence to its long–run value only after at least a decade from the increased trade exposure. The studies by Autor et al. (2013), Autor et al. (2014) and Acemoglu et al. (2016) conclude that even after more than a decade from the increased trade exposure to China the US labor market does not show signs of recovery in manufacturing jobs yet.

What can be learned from this paper is a simple theory of how unemployment, employer–to–employer reallocations and wage dynamics evolve during a trade–induced adjustment of the labor market. This paper adapts the Mortensen (2010) model of frictional labor market with homogeneous workers who search on–the–job (henceforth OJS) and wage bargaining to obtain a tractable analysis of the transitional dynamics, and extends the original setup to accommodate firm entry and endogenous job destruction. The resulting model of the labor market is then incorporated within the Melitz (2003) model of trade with heterogeneous firms making forward looking entry, export and exit decisions, under monopolistic competition and increasing returns to scale. In equilibrium, more productive firms pay better wages, hire more workers and select into the export market. The endogenous job destruction caused by trade–induced firm exit moves the labor market away from its stable allocation. The excess of unemployment and the lower probability to find a job propagate to future periods. This triggers a full dynamics of firm employment, vacancies and wages.

The model predicts short–run welfare costs from trade. This is due to the exit of less productive firms, causing higher unemployment, lower probability to find a job and lower wages; but this is not the only channel. Welfare decreases also because the allocation of workers across firms is not efficient when the probability to find a job is excessively low. Relatively low productivity firms employ too many workers, and this implies lower average revenue and profit than in the long–run with a consequent dampening of firm entry. This short–run allocation of the labor market is unstable. Over time, firms which become exporters reach their optimal employment size by hiring unemployed workers and poaching employees from less productive firms. This decreases unemployment, increases the probability to find a job, puts pressure on wages and fosters a more efficient allocation of workers across firms. In the long–run the initial job loss is offset, the average wage across workers and the average profit across firms are higher than before the trade liberalization, and fewer but larger firms populate the industry. Welfare gains from trade are now unambiguous.

The first contribution of this paper is thus to provide a dynamic analysis of the impact of trade on the labor market which is consistent with the salient observed patterns of the labor market adjustment but is also simple enough to be characterized in closed form; a feature which is not common to the few and recent related papers. The dynamics of the labor market is summarized by the evolution of two sufficient statistics which can be directly observed in the data: the unemployment rate and the duration of the unemployment spell (as the inverse of the job finding probability). The model is able to rationalize both the fear and the hope regarding international trade. The concerns about a worsening of the labor market conditions in the short–run are well taken. Nevertheless, this is not a reason to protect the economy from international trade, but rather to seek for a better functioning domestic labor market. In fact, the long–run level of unemployment is not affected by trade, the average wage is positively affected by a trade–induced increase in productivity and the speed of the transition does not depend on either trade or the output market, but only on the search and matching frictions.

The second contribution of this paper is to provide a trade model in which workers search for jobs, both when they are unemployed and employed. With the only exception of Fajgelbaum (2013), which I will briefly discuss in the following, there is no other trade model which accounts for this feature. In contrast, there is robust evidence of this reallocation mechanism in the data and in particular during the response of the labor market to a trade liberalization.[2] Furthermore, OJS is not only important because of its empirical relevance, it also introduces a key competition mechanism which is a distinctive feature of this trade model. Because of OJS, firms which offer a higher wage have better chances of hiring a worker and not losing a current employee. This channel, which is absent when only unemployed workers are assumed to search, implies that firms face an upward sloping labor supply. Thus a labor demand which is positively correlated with firm productivity (as it is common in the literature) yields a market clearing equilibrium with wage dispersion, in which more productive firms pay better wages and employ more workers.

Thanks to a dynamic analysis of the labor market with OJS, the model can also offer a deeper explanation of the mechanisms through which the export status, the wage, the employer–to–employer worker reallocations and the employment dynamics are linked during the adjustment of the labor market in response to increased trade exposure. Amiti and Davis (2012) show that exporters adjust employment faster than non–exporters, low productivity firms cut wages at the impact and then increase them during the transition. Molina and Muendler (2013) provide evidence that firms that become exporters poach workers from other firms. Davis et al. (2013) find that larger firms fill their vacancies faster, and Felbermayr et al. (2014), show that this finding extends to exporters. These patterns are captured by the model. Wages respond to the probability of finding a job which is excessively low at the impact and increases over the transition under the pressure of more workers reallocating toward more productive firms which become exporters. And the competition in the labor market introduced by OJS explains why exporters hire relatively more from other firms and overcome labor market frictions faster than non exporters.

This paper is related to a number of strands of the literature, including theories of the effect of trade on the labor market, search models of the labor market, general equilibrium trade models with transitional labor market dynamics. The approach in this paper belongs to the line of research

[2] Hall and Krueger (2012) show that about 40% of U.S. employed workers are searching while employed. Bjelland et al. (2011) show that 30% of separations each quarter are employer-to-employer reallocations. Fallick and Fleischman (2004) document that the number of monthly employer-to-employer reallocations transitions is twice as high as the flows of workers from employment to unemployment, and that nearly two-fifths of new jobs are due to employer changes. In a study on the response of the Swedish labor market to trade liberalization, Davidson et al. (2012) find that within ten years after the policy implementation 34% of workers were observed in at least two different firms, the median of on–the–job movers per firm is of 30 workers and only 3% of firms do not have on–the–job movers.

on the effect of trade on the labor market which focuses on homogeneous workers who are paid differently by heterogeneous firms because of labor market frictions.[3] In particular, other frameworks which introduce labor market frictions in a Melitz type trade model include Helpman and Itskhoki (2010), Helpman et al. (2010) and Felbermayr et al. (2011) based on wage bargaining, or Egger and Kreickemeier (2009), Amiti and Davis (2012) and Davis and Harrigan (2011) whose approach is based on fair wages. However, these models offer a steady state analysis and do not address the adjustment of the labor market. Furthermore, these models do not consider OJS, hence, despite the presence of labor market frictions, firms do not compete in the labor market. Thus, the OJS channel through which firms face an elastic labor supply represents a major departure of this paper from the previous trade models.

The theory of labor market with OJS has been developed by Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002) under wage posting and then extended by Mortensen (2010) to accommodate wage bargaining. Cahuc et al. (2006) have shown that the former source of employment plays a major role to explain the wage determination, which makes the OJS setup a more suitable candidate for explaining worker reallocation and wage dynamics, than models in which only unemployed workers search. My main contribution relative to this line of research is to embed OJS in an industry equilibrium which features monopolistic competition, endogenous firm entry and trade–induced endogenous job destruction. Moreover, I design a labor market equilibrium in which matching takes one period and prospective incumbent firms are endowed with the same number of vacancies before entry. This fits the ex–ante uncertainty upon entry of the Melitz's framework but it also allows the dynamics of the labor market to remain tractable in closed form; which is not the case in most of the search models of the labor market.[4]

This work complements recent quantitative studies on the frictional response of the labor market to trade liberalization over time; such as Dix-Carneiro (2014), Cacciatore (2014) and Cosar et al. (2013).[5] In comparison, this paper provides consistent predictions on the dynamics of unemployment, firm level employment and wages but with a richer analysis of employer–to–employer reallocations based on OJS; which is not considered in these setups. Moreover, the simulations of these quantitative models confirm the possibility of welfare losses during the transition. The simpler approach in this paper has the advantage of providing an analytical characterization of the welfare

[3] An alternative line considers heterogeneous workers and assortative matching, see Yeaple (2005), Burstein and Vogel (2012) and Sampson (2014). As already pointed out, the focus of this paper is on the reallocation within sector and across firms of workers with the same characteristics.

[4] The model in this paper does not feature aggregate uncertainty. Other recent solution methods for dynamic search models of the labor market accommodate aggregate uncertainty but do not allow for firm entry and exit; see as an example Menzio and Shi (2010) for directed search or Moscarini and Postel-Vinay (2012) for random search.

[5] Other studies in the macro–literature, such as Schaal (2012) and Kaas and Kircher (2015), develop dynamic models of the labor market with heterogeneous firms, but they do not consider international trade integration.

dynamics, showing that trade induces short–run costs (as well as long–run gains) without relying on a particular calibration.[6]

My work connects more closely to a recent working paper by Helpman and Itskhoki (2015), which extends the earlier work Helpman and Itskhoki (2010) to a dynamic setup, and to the work by Fajgelbaum (2013), which is the only other paper introducing OJS in a trade model. In Helpman and Itskhoki (2015) a firm which finds its employment below the optimal level after a trade liberalization immediately hires from the pool of unemployed workers to reach the optimal level. Long–run gains from trade are immediately realized at the impact, thus this framework does not predict short–run welfare losses.[7] Instead, in my approach the hiring success of a firm depends on the wage the firm offers, because of OJS. This competition in the labor market smooths employment and wage dynamics with a repercussion on the dynamics of aggregate unemployment and welfare. Fajgelbaum (2013) studies the effect of labor market frictions on the age at which firms invest in a foreign market. He shows that the frictions to job–to–job mobility affect income and employment, more than frictions on unemployment–to–employment flows. Hence, this study stresses the importance of OJS when analyzing the impact of trade. Nevertheless, the model provides a steady state analysis of the labor market, thus it does not address the adjustment of the labor market over time. In contrast, the work I present in this paper is the first to introduce OJS in the trade literature to the aim of explaining the short–run welfare costs and the transitional dynamics of the labor market in response to an increased exposure to trade.

The remainder of the paper is organized in four sections. Section 2 outlines the output market and the labor market. The dynamics of the labor market is presented in Section 3. Section 4 characterizes the open economy general equilibrium. In Section 5 I discuss the effect of an increased trade exposure on unemployment, wages and welfare. Section 6 concludes. The proofs of the analytical results are reported in the appendix.

2 Setup of the model

There are two countries, a domestic and a foreign economy. In each country a continuum of single product monopolists produce varieties of an horizontally differentiated good which can be traded between countries. Time is discrete and indexed by $t = 0, 1, 2, \dots$.

[6] In a recent working paper Felbermayr et al. (2014) develop a dynamic industry trade model with frictional labor market, directed search and firm heterogeneity (in productivity and age) to assess the impact of trade on inequality in Germany. Their model predicts that after a trade liberalization wages increase because of a sudden increase in average productivity. Thus, no conclusions can be drawn on the possibility that trade induces welfare losses.

[7] Although during the transition labor is mis–allocated among incumbents, with too many workers in low productivity firms as it is also predicted by my framework.

Endowments. The domestic and foreign countries are populated by a continuum of identical workers measured as N and N^*, respectively. Each worker is endowed with one unit of labor that he/she is willing to rent to firms inelastically in exchange for a given wage. Workers are immobile between countries.

Preferences. Consumption is allocated over a continuum of varieties indexed by i in the set of varieties available in the market Ω. In both countries preferences are represented by a C.E.S. utility function. Consumption allocation can be described by means of an aggregate good Q_t and a consumption based price index P_t, such that P_tQ_t is total expenditure by domestic consumers on all varieties (domestic and foreign produced) sold in the domestic market:

$$Q_t = \left[\int_\Omega q_t(i)^{\frac{\sigma-1}{\sigma}}\,di\right]^{\frac{\sigma}{\sigma-1}} \quad , \quad P_t = \left[\int_\Omega p_{qt}(i)^{1-\sigma}\,di\right]^{\frac{1}{1-\sigma}} \quad ,$$

where $q_t(i)$ and $p_t(i)$ are respectively quantity and price of the variety i and $\sigma > 1$ is the elasticity of substitution between any two varieties. There is no means to store wealth, therefore consumers do not transfer consumption across periods.

A domestic firm producing variety i sells the quantity $q_t(i)$ in the domestic market at a price $p_{qt}(i)$ and eventually exports the quantity $q_t^*(i)$ in the foreign market at a price $p_{qt}^*(i)$. A foreign producer of variety $j \neq i$ sells $\tilde{q}_t^*(j)$ units in the foreign market at a price $p_{\tilde{q}t}^*(j)$ and exports to the domestic market $\tilde{q}_t(j)$ units at a price $p_{\tilde{q}t}(j)$. The aggregate demand for a domestic variety i and an imported variety j both sold in the domestic market is given by:

$$q_t(i) = P_t^\sigma Q_t \cdot p_{qt}(i)^{-\sigma} \quad , \quad \tilde{q}_t(j) = P_t^\sigma Q_t \cdot p_{\tilde{q}t}(j)^{-\sigma} \tag{1}$$

The aggregate demand in the foreign market is $q_t^*(i) = P_t^{*\sigma} Q_t^* \cdot p_{qt}^*(i)^{-\sigma}$ and $\tilde{q}_t^*(j) = P_t^{*\sigma} Q_t^* \cdot p_{\tilde{q}t}^*(j)^{-\sigma}$. In what follows I consider the case of symmetric countries, such that $N^* = N$, $P_t^* = P_t$, $Q_t^* = Q_t$, and I outline the equilibrium for the domestic economy only.

Technology. Production employs labor according to a linear technology parameterized by the productivity $a > 0$. Firms are heterogeneous in terms of productivity. There exists an exogenous distribution of productivity $T(a)$. Every firm is endowed with one idiosyncratic realization of productivity and produces one variety only. The production function of a domestic firm endowed with productivity a is:

$$y(a, l_t) = a l_t \tag{2}$$

where l_t is employment at time t and $y(a, l_t)$ is output.

Exports are associated with an additional ad–valorem cost. In order to sell one unit of good in the export market a firm ships $\tau \geq 1$ units. Let the indicator function $\mathbf{1}(a) = \{0, 1\}$ denote the exporter status: $\mathbf{1}(a) = 1$ indicates that the firm is an exporter, otherwise $\mathbf{1}(a) = 0$. Market clearing at the firm level and feasibility of production imply:

$$q_t(a) + \mathbf{1}(a)\tau q_t^*(a) = y(a, l_t) \tag{3}$$

where I anticipate that the export decision is ultimately a function of firm productivity. Notice that since serving an export market requires τq_t^* units of additional production, if an exporter firm demands l_t^d employees to serve the domestic market then additional $l_t^x = \tau^{1-\sigma} l_t^d$ employees are demanded to serve the export market. This yields a total employment of $l_t = \left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right) l_t^d$.

Revenue. Firms maximize profit in each destination, based on consumer demand (1), technology (2) and market clearing (3). Equating marginal revenues across markets yields: $p_{qt}^* = \tau p_{qt}$. The difference in prices translates into differences in demand in the two destination markets: $q_t^* = \tau^{-\sigma} q_t$. Inverse demand and market clearing yield the firm's total revenue as a function of productivity and employment:

$$r(a, l_t) \quad = \quad \left[\left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right) P_t^{\sigma} Q_t\right]^{\frac{1}{\sigma}} (a l_t)^{\frac{\sigma-1}{\sigma}} \tag{4}$$

Notice that total revenue can also be written as $r(a, l_t) = \left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right) r^d(a, l_t)$ where $r^d(a, l_t) = (P_t^{\sigma} Q_t)^{\frac{1}{\sigma}} \left(a l_t^d\right)^{\frac{\sigma-1}{\sigma}}$ is the revenue from domestic sales.

Wage. Firms employ many workers and when making their wage offer they anticipate the outcome of a bargaining with the marginal worker, as discussed in Stole and Zwiebel (1996). In the case that the two parties break the negotiation, the firm loses the profit from the marginal worker in the current period $\frac{\partial r(a,l_t)}{\partial l_t} - \frac{\partial w(a,l_t)}{\partial l_t} l_t - w(a, l_t)$, and the worker loses the opportunity to earn a wage $w(a, l_t)$. Assuming symmetric bargaining power between the two parties, the wage is obtained as the particular solution to the ordinary differential equation which equates the two option values:

$$w(a, l_t) = \frac{\sigma - 1}{2\sigma - 1} \frac{r(a, l_t)}{l_t} \tag{5}$$

which I will refer to as the *wage equation*.

2.1 Industry

Potential entrants pay a fixed cost $f_e > 0$ once to enter the market. This is a sunk investment which prospective entrants make before they know how efficient they will be on the market. Firms realize their productivity in the following period and they might decide to actually enter the market. In this case f_e pays for the cost of hiring the workers the firm will start matched with, otherwise this initial investment provides no value. Incumbent firms pay a fixed cost $f_p > 0$ each period to enable production. Firms that decide to export pay a fixed cost for operating in the export market $f_x > 0$. All fixed costs are in nominal terms.[8]

Because of fixed costs, firms might not break even and make losses. For this reason, at the beginning of every period t incumbents decide to compete or exit. Exit provides zero value. Hence, all firms that are sufficiently productive to realize a positive lifetime value remain, others choose optimally to exit; I refer to this choice as *endogenous exit.* Let the productivity cutoff a_t^{in} be the maximum threshold below which firms endowed with productivity $a < a_t^{in}$ exit at time t. In addition to endogenous exit, firms can be forced to exit because of an idiosyncratic destructive shock which occurs with exogenous probability $\delta_f \in (0,1)$; I describe such an event as *exogenous exit.*

Payments occur at the end of a period and total profit is then allocated to finance the entry of new firms. The mass of incumbent firms in a given period M_{t+1} consists of all previous incumbents endowed with a productivity that is not lower than the current cutoff and did not exit because of an exogenous shock, plus new entrants which paid the entry cost in the previous period E_t and realize a productivity that is not lower than the current cutoff:

$$M_{t+1} = \left[1 - T\left(a_{t+1}^{in}\right)\right] E_t + (1-\delta_f)\, \mu_{t+1}^{in} M_t \tag{6}$$

Equation (6) is the *law of motion for the mass of firms* in the industry, where $\mu_{t+1}^{in} = \frac{1-T\left(a_{t+1}^{in}\right)}{1-T\left(a_t^{in}\right)}$ for $a_{t+1}^{in} \geq a_t^{in}$ and 1 otherwise.

2.2 Labor market

Search is costly for firms and time consuming for workers: firms pay a cost $k > 0$ for each vacancy posted, unemployed workers send 1 job application per period, employed workers search by sending $\phi > 0$ job applications per period. Matching is random and takes one period: vacancies posted in period $t-1$ are matched with job applications sent in period $t-1$, leading to job reallocation in period t. The matching technology is assumed to be homogeneous of degree 1 for vacancies and job applications, as documented in the empirical literature. In particular, I assume the matching technology proposed by Ramey et al. (2000) which has the advantage of a matching probability bounded in the unit interval for both workers and firms, one being the complement of the other. Hence, I call $x_t \in (0,1)$ the probability that a worker matches with a firm in period t, while the probability that a vacancy is visited by a worker is $1 - x_t$. In this context, I will refer to the labor market in time t as *tighter* when the probability x_t is higher.

The state of the labor market consists of the number of unemployed workers u_t and the job finding probability x_t; which yields the state vector $z_t = \{u_t, x_t\}$. Two cumulative density functions (c.d.f.)

[8] This assumption differs from Melitz (2003), where firms employ additional workers as a fixed factor of production. In the context of this paper, more productive firms will pay higher wages and a characterization of fixed costs in terms of employment would have made firms heterogeneous in terms of their fixed costs structure.

characterize the labor market allocation conditional on the state of the labor market: $G(w; z_t)$ is the share of employed workers who accept a wage lower than w or equal; $F(w; z_t)$ is the share of wage offers which will be accepted by a worker earning a wage lower than w or equal. These two c.d.f. will be determined endogenously as outcome of an equilibrium which features wage dispersion; as it will be discussed in the next section.

Timing. At the end of every given period $t-1$ there are u_{t-1} unemployed workers and $n_{t-1} = N - u_{t-1}$ workers are employed. At the beginning of period t the productivity cutoff a_t^{in} is understood, and this determines firm exit and firm entry decisions. When a firm exits its jobs are destroyed. Hence, a share δ_f of jobs is destroyed because of exogenous firm exit; and a share $\varepsilon_t \in [0, 1)$ of jobs is destroyed because of endogenous firm exit (the variable ε_t is an equilibrium outcome to be determined). In addition, jobs at firms which do not exit can be destroyed by an exogenous shock which occurs with probability $\delta_j \in (0, 1)$. Job creation consists of the vacancies posted in the previous period by both new firms which made a successful entry and previous incumbents which did not exit.

Following firm entry and exit, the labor market opens. Vacancies and job applications are matched. Firms make offers simultaneously, without recall and, since discrimination between identical workers is not allowed, they offer the same wage to current employees and workers they match with on the market. Workers are either unemployed or they have in hand the wage offer from the employer to which they have been matched in the previous period. Workers who receive an offer on the market compare it with their current status and decide whether to accept the offer or not. Worker reallocation occurs, leading to the new allocation of workers: u_t unemployed workers, n_t employed workers. Before the current period ends, firms post vacancies for the next period.

Worker reallocation. Workers are infinitely living and participate in the labor market to the end of maximizing their lifetime discounted income. A reservation wage for unemployed workers exists such that the value of being employed at this reservation wage makes workers indifferent with respect to remaining unemployed. It can then be shown that rational unemployed workers accept every wage which is not lower than this reservation wage. For employed workers, rejecting an offer by employers met on the market does not lead to unemployment as they remain matched with their previous employer at the current wage.[9] The patterns of worker reallocation can be understood by looking at the share of employed workers who separate from their current employer and at the firm hiring success rate per vacancy posted. A worker and a firm which offers a wage w separate either because of an exogenous job destruction shock or because the worker accepts a better wage offer. The same probability of separation applies to all firm employees, and if thought of as a continuous

[9]The Bellman equations for the worker search problem are discussed in the appendix.

measure then the share of employees who separate from a given firm which offers a wage w in the market at time t is:

$$s(w; z_t) = \delta_j + (1 - \delta_j)\,\phi x_t \left[1 - F(w; z_t)\right] \tag{7}$$

which I will refer to as *separation rate* and where $x_t\left[1 - F(w; z_t)\right]$ is the probability that a worker receives a wage offer better than w. The probability that a worker visits a firm's vacancy is $1 - x_t$ but this possible match becomes a new hiring if and only if the worker accepts the wage offer. The probability that a vacancy is filled for a firm which offers a wage w is given by the *hiring rate*

$$h(w; z_t) = (1 - x_t)\left(\frac{u_{t-1} + \phi n_{t-1}(\delta + \varepsilon_t)}{u_{t-1} + \phi n_{t-1}} + \frac{\phi n_{t-1}(1 - \delta - \varepsilon_t)}{u_{t-1} + \phi n_{t-1}} G(w; z_t)\right) \tag{8}$$

where the total number of job applications is $u_{t-1} + \phi n_{t-1}$, of which $u_{t-1} + \phi n_{t-1}(\delta + \varepsilon_t)$ sent by workers who have unemployment as outside option, and $\phi n_{t-1}(1 - \delta - \varepsilon_t)$ by workers who might remain matched with their current employer; then the parameter $\delta = \delta_f + (1 - \delta_f)\delta_j$ accounts for the two sources of exogenous job destruction. Notice that although all firms meet a worker with the same probability, firms which pay a higher wage have a larger probability of hiring a worker and they are less likely to separate from current employees. This is a distinctive feature of OJS, as it clearly applies if and only if $\phi > 0$.

The reallocation of workers occurs immediately after matching and determines the employment for every firm in the current period. The employment of a firm matched with l_{t-1} employees, which posted v_{t-1} vacancies and offers a wage w in the labor market at time t, is given by:

$$l_t = \left[1 - s(w; z_t)\right] l_{t-1} + h(w; z_t)\, v_{t-1} \tag{9}$$

which is implied by the balance between separations $s(w; z_t)\, l_{t-1}$ and hirings $h(w; z_t)\, v_{t-1}$. I will refer to equation (9) as the *law of motion for employment.*

OJS induces competition in the labor market. An incumbent firm makes $(1 - \delta_j)\, l_{t-1}$ job offers at a wage w to its previous employees whose jobs were not destroyed, and $(1 - x_t)\, v_{t-1}$ job offers to workers with whom the firm matches in the market. From a total of $(1 - \delta_j)\, l_{t-1} + (1 - x_t)\, v_{t-1}$ wage offers, the number of accepted job offers is given by the law of motion for employment (9). The employment of a new entrant, which is endowed with f_e/k vacancies financed with the entry cost and offers the same wage w to $(1 - x_t)\, f_e/k$ workers is $h(w; z_t)\, f_e/k$. Since these two firms pay the same wage w, and all workers receive offers from the market randomly, the acceptance rate per offer is the same for the two firms; which yields:

$$\frac{1 - s(w; z_t)}{h(w; z_t)} = \frac{1 - \delta_j}{1 - x_t} \tag{10}$$

This result establishes that the probability of hiring a worker who visited a firm vacancy $\frac{h(w;z_t)}{1-x_t}$ is equal to the probability that a firm employee does not accept the job offer of another employer $\frac{1-s(w;z_t)}{1-\delta_j}$. Thus, equation (10) describes the competition firms face in the labor market.

Labor market dynamics. The *law of motion for unemployment* is given by the balance between previously unemployed workers who do not find a job plus workers whose jobs are destroyed in the current period and do not receive job offers:

$$u_{t+1} = (1 - x_{t+1})\, u_t + (1 - \phi x_{t+1})\, (\delta + \varepsilon_{t+1})\, (N - u_t) \quad (11)$$

Unemployment is a decreasing function of the current job finding probability and rises with endogenous job destruction. The system of separation rate (7), hiring rate (8) and the competition condition (10) evaluated at the lower bound of the wage support w_{0t}, such that $F(w_{0t}; z_t) = G(w_{0t}; z_t) = 0$, allows the job finding probability to be written as:

$$x_{t+1} \;=\; \frac{(1 - \delta - \varepsilon_{t+1})\,(N - u_t)}{u_t + \phi(N - u_t)} \quad (12)$$

Equation (12) yields the *law of motion for the job finding probability.* The probability to find a job in a given period is decreasing in the share of jobs destroyed in the current period. The system of difference equations (11) and (12) governs the dynamics of the labor market, given the share of jobs destroyed by endogenous exit ε_{t+1}, which is determined in general equilibrium as discussed in the following section. In steady state no endogenous firm exit occurs, since the cutoff productivity does not change by definition. It follows that steady state unemployment $u = \frac{\phi\delta^2 N}{1-2\delta+\phi\delta^2}$ and job finding probability $x = \frac{1-2\delta}{\phi(1-\delta)}$ exist and are unique.[10]

Out of the steady state, an unanticipated policy implementation causing endogenous firm exit determines an endogenous job destruction. The variable $\varepsilon_{t+1} > 0$ jumps to a strictly positive value at period of policy implementation. Unemployment jumps to a higher level than in the steady state $u_i > u$ and job finding probability jumps to a lower level than in the steady state. The following dynamics toward the unique steady state is disciplined by the system (11)–(12) evaluated in the absence of further endogenous shocks $\varepsilon_{t+1} = 0$. Substituting for x_{t+1} in the law of motion of unemployment (11) reveals that the dynamics of unemployment is autonomous and the future level of unemployment is a strictly positive, increasing and convex function of the current level. Figure (1) shows the dynamic equation for unemployment. There is one non–trivial and stable steady state. The level of unemployment induced by endogenous firm exit is unstable. Starting from any $u_i > u$ the transition of unemployment toward its steady state is monotonic $u_{t+1} \leq u_t$. Future level of job finding probability depends only on the current unemployment level. Hence, the dynamics of the job finding probability is monotonic and increasing $x_{t+1} \geq x_t$.[11]

Firm vacancy posting. The number of vacancies is chosen at the end of the period as the outcome of an inter-temporal optimality problem in which the firm maximizes its value. The endogenous

[10]Notice that data on unemployment rate u/N and duration of unemployment spell $1/x$ are sufficient to calibrate the two key parameters of the labor market, δ and ϕ.

[11]See the appendix for a deeper analysis of the stability.

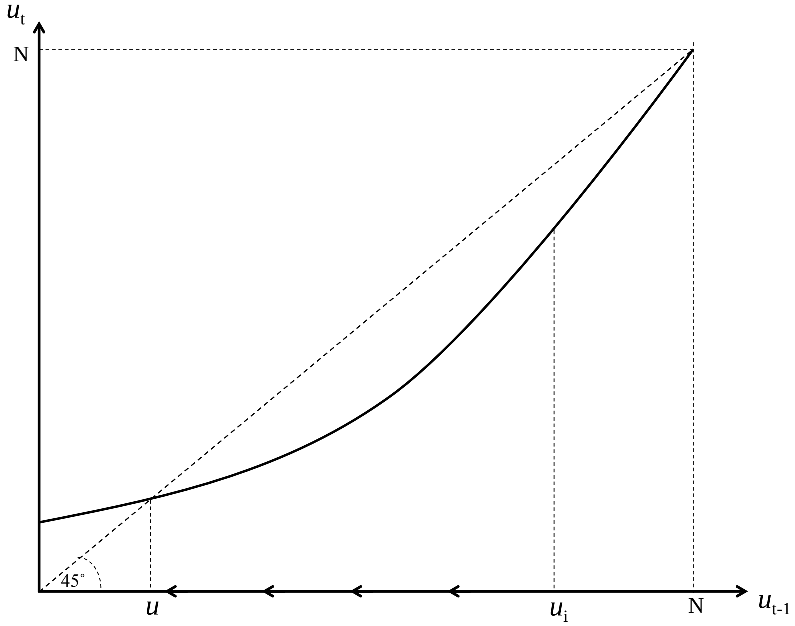

Figure 1: Dynamics of the unemployment rate

aggregate state of the firm's problem consists of job finding probability and unemployment $z_t = \{x_t, u_t\}$, while productivity a and firm employment l_t are the individual state variables. The number of vacancies $v_t > 0$ is the control variable. The value of the firm is given by:

$$
\begin{aligned}
V(a, l_t; z_t) &= \max_{v_t > 0} \{r(a, l_t) - w(a, l_t) l_t - kv_t - f_p - \mathbf{1}(a) f_x + \\
&+ (1 - \delta_f) V(a, l_{t+1}; z_{t+1})\}
\end{aligned} \tag{13}
$$

subject to the law of motion for employment at the firm level (9), unemployment (11) and job finding probability (12); where the revenue satisfies (4), the wage equation (5) holds and the firm does not anticipate future policies causing endogenous exit.

3 Equilibrium in the labor market

In this section I define a frictional labor market equilibrium with wage dispersion. Differently from other models of the labor market which feature OJS, such as Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002), this model characterizes a monotonic increasing relationship between productivity, wage and employment. Thus, the labor market exhibits a *rank–preserving equilibrium* in which employer–to–employer reallocation is always efficient, as it is discussed in Moscarini and Postel-Vinay (2012). This is the essential feature through which a labor market with OJS has an impact on the general equilibrium of the model. Many other elements of existing OJS frameworks are not considered (such as wage posting, multiple offers with recall, Markov contingent contracts and aggregate uncertainty), in exchange for the simplest setup which provides an analytical solution of the dynamics out of the steady state and accommodates firm entry and exit.

DEFINITION. A *monotone increasing wage dispersion equilibrium with firm entry* consists of a wage–productivity assignment function $\omega(a;z_t) : [a_t^{in}, \infty) \rightarrow [w_{0t}, w_{1t}]$, a firm profit $\pi(a, l_t; z_t)$: $[a_t^{in}, \infty) \times (0, \infty) \rightarrow [0, \infty)$, a hiring rate $h(w; z_t) : [w_{0t}, w_{1t}] \rightarrow (0, 1)$, a wage support w_{0t} : $G(w_{0t}; z_t) = 0$ and $w_{1t} : G(w_{1t}; z_t) = 1$ such that:
(i) firm wage satisfies the bargaining (5) and is increasing in productivity $d\omega(a;z_t)/da \geq 0$
(ii) firm vacancies solve the inter–temporal problem (13)
(iii) deviations from the implied firm employment are not profitable $\partial\pi(a, l_t; z_t)/\partial l_t = 0$
(iv) the marginal firm which enters the market makes zero value.

A policy for vacancies $\vartheta(a, l_t; z_t)$ which solves the firm problem (13) satisfies the necessary first order condition for an interior solution:

$$(1 - \delta_f)\, h\, (w(a, l_{t+1}; z_{t+1}); z_{t+1})\, J\, (a, l_{t+1}; z_{t+1}) = k \tag{14}$$

where $J\,(a, l_t; z_t) = \frac{\partial V(a,l_t;z_t)}{\partial l_t}$ is the value of the marginal job at time t. The choice of an optimal number of vacancies is unconstrained due to the linear cost of vacancy posting. However, in a monotone increasing wage dispersion equilibrium there is one feasible choice for the number of vacancies. In fact, if there exists a monotonic and increasing relationship between productivity and wage, then the wage equation (5) implies that every given level of productivity $a \geq a_t^{in}$ maps into one and only one employment level l_t. A new entrant endowed with productivity a at time t employs $l_t = h\,(w\,(a, l_t)\,; z_t)\, f_e/k$ workers and this has to be the employment of every firm endowed with the same productivity level; which also shows that more productive firms employ more workers because they pay higher wages. Substituting in the law of motion for employment (9) yields the policy for vacancies:

$$\vartheta(a, l_t; z_t) = \left(\frac{1}{h\,(w\,(a, l_t)\,; z_t)} - \lambda(u_t)\right) l_t \tag{15}$$

where the function $\lambda(u_t) = \frac{(1-\delta_j)[u_t+\phi(N-u_t)]}{u_t-[(1-\delta)-\phi](N-u_t)}$ decreasing in u_t shows that firms post more vacancies when unemployment is high, conditional on productivity and employment.[12] The profit of a firm endowed with productivity a which employs l_t workers when the state of the labor market is z_t is given by $\pi(a, l_t; z_t) = r(a, l_t) - w(a, l_t)l_t - k\vartheta(a, l_t; z_t) - f_p - \mathbf{1}(a)f_x$. The intra–temporal decision on wage (5) and the inter–temporal decision on vacancies (15) imply the level of firm employment, given productivity and state of the labor market. If firms do not have the incentive to deviate from the implied level of employment and the wage equation (5) holds then the policy for vacancies must

[12]The function $\lambda(u_t)$ is obtained after the substitution for competition in the labor market (10) and the law of motion of job finding probability (12) given that no endogenous firm exit is anticipated.

satisfy $\partial\vartheta(a, l_t; z_t)/\partial l_t = w(a, l_t)/k$. A second expression for $\partial\vartheta(a, l_t; z_t)/\partial l_t$ is given by the partial derivative of vacancies (15) with respect to current employment. The system of the two conditions yields a Bernoulli differential equation in $h(w; z_t)$. Let the firm which pays the wage w_{0t} have the minimum hiring rate h_{0t}. A solution which passes through the point $h(w_{0t}; z_t) = h_{0t}$ exists, is unique and can be derived in closed form:

$$h(w; z_t) = \frac{h_{0t}}{\alpha(z_t) + \beta(z_t)\frac{w}{w_{0t}} - (\alpha(z_t) + \beta(z_t) - 1)\left(\frac{w}{w_{0t}}\right)^{\sigma}} \tag{16}$$

where $\alpha(z_t) = \lambda(u_t)h_{0t}$, $\beta(z_t) = \frac{\sigma}{\sigma-1}\frac{w_{0t}h_{0t}}{k}$. Evaluating the hiring rate (8) at the extremes of the wage support yields $h_{0t} = (1 - x_t)(1 - \phi x_t)$ and $h_{1t} = 1 - x_t$. Then for a given level w_{0t}, the upper bound of the wage support w_{1t} is determined such that $h(w_{1t}; z_t) = h_{1t}$. The c.d.f. of wages at which workers reallocate $G(w; z_t)$ is obtained by inverting the hiring rate (8) and substituting for $h(w; z_t)$ as in the equilibrium condition (16). Then, the separation rate (7) and the condition for competition in the labor market (10) determine the c.d.f. of wage offers $F(w; z_t)$.

Notice that the hiring rate, and indeed the separation rate and the c.d.f. of the wage distributions, depend neither on firm productivity nor on the cutoff productivity. Thus, this labor market structure implies that the only channel through which firms compete on the labor market is wage. As a consequence, all firms face the same increasing labor supply curve, which is given by $h(w; z_t)f_e/k$ as the number of workers a firm employs when offering a wage w. Heterogeneity in productivity does not affect the labor supply, instead it affects wage determination as a demand shifter. The system of revenue (4) and wage equation (5) yields the labor demand of the firm. The clearing of labor demand and supply at the firm level determines the assignment of wage to productivity.[13] An explicit solution can be derived for the inverse function $a = \omega^{-1}(w; z_t)$, which solves:

$$\left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1} = \frac{h(w; z_t)}{h(w_{0t}; z_t)}\left(\frac{w}{w_{0t}}\right)^{\sigma} \tag{17}$$

where the monotonicity of the wage productivity assignment implies that the firm endowed with the cutoff productivity pays the wage at the lower bound of the support $\omega(a_t^{in}; z_t) = w_{0t}$.

Substituting the vacancy policy (15) in the expression for profit shows that more productive firms make higher profits, for two reasons. They make a greater profit per worker and they employ more workers. Substituting (17) in (16) and rearranging to explicit for the labor payroll then substituting for $\frac{\sigma}{\sigma-1}w(a, l_t)l_t$ yields the profit of a firm as a function of firm's productivity $a \geq a_t^{in}$

$$\pi(a; a_t^{in}) = \left[\left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1} - 1\right] f_p - \mathbf{1}(a)f_x \tag{18}$$

[13] Let p_t and l_t^d be respectively the price on the domestic market and the employment required to serve the domestic market for a firm endowed with productivity $a \geq a_t^{in}$, producing al_t^d units of output for the domestic market and making a domestic revenue $r_t^d(a)$. Then the wage equation (5) implies $p_t = \frac{2\sigma-1}{\sigma-1}\frac{w}{a}$, where w is the wage paid by the firm endowed with productivity a. Let r_{0t} be the revenue on the domestic market made by a firm endowed with productivity a_t^{in} then the relative productivity satisfies $\frac{a}{a_t^{in}} = \frac{w}{w_{0t}}\left(\frac{r_t^d(a)}{r_{0t}}\right)^{\frac{1}{\sigma-1}} = \frac{w}{w_{0t}}\left(\frac{1}{1+\mathbf{1}(a)\tau^{1-\sigma}}\frac{r_t(a)}{r_{0t}}\right)^{\frac{1}{\sigma-1}}$.

Notice that the profit of a firm does not depend on the state of the labor market. This implies that firm entry, exit and export decisions can be evaluated independently on the transitional dynamics of the labor market.[14] Thus, the value of a firm endowed with productivity a is $V\left(a; a_t^{in}\right) = \pi(a; a_t^{in})/\delta_f$. The maximum productivity level below which the value of a firm is negative solves $\pi(a_t^{in}; z_t) = 0$; and if there is selection in the export market then a firm endowed with the productivity cutoff does not export. This condition allows the lower bound of the wage support to be determined:

$$w_{0t} = \frac{\sigma - 1}{\sigma} \left(\frac{f_p + f_e}{(1 - x_t)(1 - \phi x_t) f_e} - \lambda(u_t) \right) k \tag{19}$$

This completes the characterization of the unique wage dispersion equilibrium with firm entry.

In this model the transitional adjustment of the labor market is determined in closed form by the system of dynamic equations (11)–(12) and it propagates to several firm–level dimensions. Revenue, wage, employment, separation and hiring rates evolve over time depending on the state of the labor market, hence on unemployment and job finding probability. Nevertheless, these contributions cancel out in the expression of firm profit and this makes the determination of the productivity cutoff identical to Melitz's industry equilibrium. Despite the simplicity, this framework delivers novel predictions: employment, wages and their distribution across firms and workers evolve over time in response to job destruction which follows trade–induced firm exit. In contrast with Melitz (2003) the aggregate productivity (and in turns welfare) exhibits a transitional dynamics in response to increased trade exposure; as will be discussed in the next section.

4 General equilibrium

In this section I describe the determination of the general equilibrium, in two steps. First, export participation and the aggregate properties of the model are discussed, by looking at a representative firm which has employment equal to the average employment per firm in the economy. Then the model is closed and a distinction between the long–run and the short–run equilibrium is discussed.

4.1 Aggregation

The profit of a firm in the domestic market is $\pi^d(a; a_t^{in}) = [(a/a_t^{in})^{\sigma-1} - 1] f_p$, while the profit in the export market is $\pi^x(a; a_t^{in}) = (a/a_t^{in})^{\sigma-1} \tau^{1-\sigma} f_p - f_x$. Since the value of a firm is proportional to firm profit, the indifference condition $\pi^x(a_t^x; a_t^{in}) = 0$ implies $a_t^x = \tau (f_x/f_p)^{\frac{1}{\sigma-1}} a_t^{in}$ which determines the productivity cutoff above which firms select into the export market.

[14] In fact, firm profit in equilibrium (18) has the same expression as the one in Melitz (2003). This might sound at the same time reassuring (since the setup of the industry is identical) and surprising (as this model features a wage dispersion equilibrium with unemployment).

The revenue is a linear function of profit and employment.[15] It follows that a firm which makes the average revenue $\bar{r}_t = R_t/M_t$, also makes the average profit $\bar{\pi}_t$ and employs the average number of workers $(N - u_t)/M_t$. Let $\bar{a}_t$ be the productivity of the firm with average employment, such that $h(\omega(\bar{a}_t; z_t); z_t) f_e/k = (N - u_t)/M_t$ is the firm employment and $\bar{w}_t = \omega(\bar{a}_t; z_t)$ is the firm wage. Moreover, notice that total labor income in the economy is $L_t = \bar{w}_t(N - u_t)$ and the wage equation (5) implies that $\bar{w}_t$ solves $L_t = M_t \times \bar{w}_t h(\bar{w}_t; z_t) f_e/k$. Therefore, $\bar{w}_t$ is the average wage across employed workers. The identity $r(\bar{a}_t; z_t) = \bar{r}_t$ jointly with the expression for profit (18), and the wage assignment (17), implies the two equalities:

$$\left(1 + \mathbf{1}(\bar{a}_t)\tau^{1-\sigma}\right)\left(\frac{\bar{a}_t}{a_t^{in}}\right)^{\sigma-1} = \frac{h(\bar{w}_t; z_t)}{h(w_{0t}; z_t)}\left(\frac{\bar{w}_t}{w_{0t}}\right)^{\sigma} = 1 + \frac{\bar{\pi}_t + \mu_t f_x}{f_p} \tag{20}$$

The condition (20) allows the average productivity $\bar{a}_t$ and the average wage $\bar{w}_t$ to be determined, given the state of the labor market $z_t = \{u_t, x_t\}$ and the pair of cutoff productivity and average profit $\{a_t^{in}, \bar{\pi}_t\}$.

Aggregate profit is used to finance the possible entry of new firms in the following period $f_e E_t = \bar{\pi}_t M_t$. Substituting in the the law of motion (6) yields the mass of firms M_t as proportional to the entry probability, where the factor of proportionality is predetermined. A second expression for the mass of firms is implied by the labor market allocation. The equilibrium hiring rate (16) yields the average employment per firm $h(\bar{w}_t; z_t) f_e/k$. Therefore, the mass of firms is determined through the definition of average employment.[16] These two relationships

$$\begin{aligned} M_t &= \left[1 - T(a_t^{in})\right]\left(\frac{\bar{\pi}_{t-1}}{f_e} + \frac{1-\delta_f}{1 - T(a_{t-1}^{in})}\right) M_{t-1} \\ M_t &= \frac{N - u_t}{h(\bar{w}_t; z_t) f_e/k} \end{aligned} \tag{21}$$

link the industry dynamics and the labor market dynamics.

Closing the model requires to determine six elements: a pair of values for the cutoff productivity and the average profit $\{a_t^{in}, \bar{\pi}_t\}$; the state vector of the labor market, which consists of unemployment and job finding probability $z_t = \{u_t, x_t\}$; then, the equilibrium conditions (20) and (21) are sufficient to determine the average wage and the mass of firms $\{\bar{w}_t, M_t\}$.

For the sake of exposition, I summarize the impact of trade by looking at three points in time: B indicates the steady state before a trade liberalization, i indicates the time of implementation of a trade liberalization policy, and A indicates the steady state after a trade liberalization.

[15] Using the wage equation (5) and the profit (18) to determine firm revenue . Details are reported in the appendix.

[16] An equivalent determination of the mass of firms is obtained the economy budget constraint. The policy for vacancies (15) implies a total cost of vacancy posting $K_t = f_e M_t - k\lambda(u_t)(N - u_t)$. Total fixed costs, with a share μ_t of exporters are $(f_p + \mu_t f_x) M_t$. Total profit is $\Pi_t = \bar{\pi}_t M_t$, and the number of potential new entrants in the next period is $f_e E_t = \Pi_t$. The budget constraint of the domestic economy is $R_t - f_e E_t = L_t + K_t + (f_p + \mu_t f_x) M_t$. Substituting for the aggregate variables yields the mass of firms as implied by the definition of average employment.

4.2 Long–run equilibrium

The long–run is a state of the economy in which: (i) the entry of firms is unbounded; (ii) the productivity cutoff and the mass of firms are in steady state; (iii) the labor market is in steady state. An unbounded entry of firms implies that the average profit of an incumbent firm is equal to the expected profit conditional on entry. Defining the function φ and its cutoff values

$$\varphi\left(a'\right) \quad = \quad \left[\int_{a'}^{\infty} a^{\sigma-1}\frac{dT(a)}{1-T(a')}\right]^{\frac{1}{\sigma-1}} \quad , \quad \varphi^{in}=\varphi\left(a_t^{in}\right) \ , \ \varphi^{x}=\varphi\left(a_t^{x}\right)$$

allows the conditional expected profit in the two markets to be written as $\bar{\pi}_t^d=\pi^d(\varphi_t^{in};a_t^{in})$ and $\bar{\pi}_t^x=\pi^x(\varphi_t^{x};a_t^{in})$. The expected profit of an incumbent firm is $\bar{\pi}(a_t^{in})=\bar{\pi}_t^d+\mu_t\bar{\pi}_t^x$, where $\mu_t=\frac{1-T(a_t^x)}{1-T\left(a_t^{in}\right)}$ is the share of exporter firms. The export indifference condition implies that ceteris paribus the expected profit is higher the lower the barriers to trade. Accounting for this comparative statics, the expected profit conditional on entry is a function of the cutoff productivity $\bar{\pi}(a_t^{in};\tau,f_x)$ decreasing in the policy parameters τ and f_x. Let $\bar{\pi}_s$ be the average profit evaluated in steady state $t=s$, either before or after the trade liberalization. Then the *zero profit condition*

$$\bar{\pi}_s=\bar{\pi}(a_s^{in};\tau,f_x) \qquad \text{for } s=B,A \tag{22}$$

yields the the equality between average profit of an incumbent firm and expected profit conditional on entry. The expected value of entry (ex–ante the realization of firm productivity) is the expected value conditional on entry $\bar{\pi}(a_s^{in};\tau,f_x)/\delta_f$ times the probability of making a successful entry $\left[1-T(a_s^{in})\right]$. In the long–run, the expected value of entry cannot be lower than the cost of entry; and, because of free entry, the expected value of entry cannot exceed the cost of entry. Therefore, in steady state a *free entry condition* holds:[17]

$$\frac{\bar{\pi}(a_s^{in};\tau,f_x)}{\delta_f}=\frac{f_e}{1-T(a_s^{in})} \qquad \text{for } s=B,A \tag{23}$$

A pair of values for the productivity cutoff and the average profit $\left\{a_s^{in},\bar{\pi}_s\right\}$ which solves (22)–(23) exists and is unique. The state vector of the labor market has a unique steady state $z_s=\{u_s,x_s\}$ which is determined by the labor market parameters only, as discussed in the third section. The equilibrium condition (20) determines a unique average wage $\bar{w}_s$ for a given average profit $\bar{\pi}_s$ and state of the labor market z_s. Average employment $h\left(\bar{w}_s;z_s\right)f_e/k$ is fixed by the equilibrium hiring rate (16). Total employment $N-u_s$ is fixed by the unemployment level. Hence, there exists a unique value for the mass of firms $M_s=(N-u_s)/[h\left(\bar{w}_s;z_s\right)f_e/k]$ which satisfies the definition of average employment. This completes the determination of the steady state long–run equilibrium.

The steady state values of unemployment and job finding probability depend only on the parameters of the labor market: size of the workforce N, exogenous job destruction δ and job applications

[17]Notice that the free entry condition can also be obtained as the unique steady state solution for the dynamics of the mass of firms (21).

by employed workers ϕ; as it is clear from the labor market dynamics (11)–(12). Comparative statics on the system (22)–(23) imply that the cutoff productivity $a_A^{in} > a_B^{in}$ and the average profit $\bar{\pi}_A > \bar{\pi}_B$ are higher in the steady state after a trade liberalization, while participating in the export market is easier $a_A^x < a_B^x$. Under mild assumptions on the exogenous productivity distribution, also the sum $\bar{\pi}_t + \mu_t f_x$ is necessarily higher in the steady state after the trade liberalization.[18] This is a sufficient condition for two results. First, after a trade liberalization there are less firms producing in the domestic market $M_A < M_B$ and they employ on average more workers. Second, the average wage is higher after the trade liberalization $\bar{w}_A > \bar{w}_B$, as implied by the equilibrium condition (20) given $u_A = u_B$ and $x_A = x_B$. This completes the steady state comparison of the economy before and after a trade liberalization.

4.3 Endogenous job destruction

An increase of the productivity cutoff causes endogenous firm exit which leads to job destruction. Previous incumbents endowed with a productivity which is lower then the new cutoff exit the market and their employees become unemployed, before the labor market opens and production starts in the upcoming period. Let $a_{t+1}^{in} > a_t^{in}$, then the number of firms that exit the market at the beginning of period $t+1$ is given by $M_{t+1}^{ex} = \left\{[T(a_{t+1}^{in}) - T(a_t^{in})]/[1 - T(a_t^{in})]\right\} M_t$. The monotonicity of the wage–productivity assignment implies that the workers who lose their jobs because of endogenous firm exit are the subset of the employed workers in the previous period who were earning a wage $\omega(a_{t+1}^{in}; z_t) > w_{0t}$ or lower. The c.d.f. of the distribution of wages across firms in a given period is a transformation of the productivity c.d.f. $\mathcal{T}(w; z_t) = [T(\omega^{-1}(w; z_t)) - T(a_t^{in})]/[1 - T(a_t^{in})]$ and its density is defined by the wage productivity assignment (17).[19] The hiring rate (16) determines firm employment, which is a continuous and increasing function of the wage. At the end of period t the number of workers employed in the firms which decide to exit the market at time $t+1$ is given by:

$$n_{t+1}^{ex} = M_{t+1}^{ex} h_{t+1}^{ex} f_e / k \quad , \quad \text{where} \quad h_{t+1}^{ex} = \int_{w_{0t}}^{\omega(a_{t+1}^{in}; z_t)} h(w; z_t) \frac{d\mathcal{T}(w; z_t)}{\mathcal{T}(\omega(a_{t+1}^{in}; z_t); z_t)}$$

is the average hiring rate among the firms that exit at time $t+1$. The share of jobs destroyed because of endogenous firm exit is:

$$\varepsilon_{t+1} = \frac{n_{t+1}^{ex}}{N - u_t} \tag{24}$$

where $\varepsilon_{t+1} > 0$ if and only if $a_{t+1}^{in} > a_t^{in}$, and $\varepsilon_{t+1} = 0$ otherwise. Condition (24) shows that for a given increase in the productivity cutoff, the larger is the share of workers employed at lower quantiles of the wage support the more severe the job destruction hits.

[18] See the discussion in the appendix.

[19] The assignment (20) defines the inverse function $a = \omega^{-1}(w; z_t)$, which is continuous and differentiable over the support $a_t^{in} \leq a < a_t^x$. In this range of productivity values, the derivative $da(w; z_t)/dw$ is positive, continuous and it can be computed analytically.

4.4 Short–run equilibrium

The short–run is a state of the economy in which: (i) the long run patterns of firm entry, exit and export participation are correctly foreseen, but (ii) the current mass of firms, the average profit and the labor market allocation are not in steady state. The time of implementation of a trade liberalization $t = i$ is the first period of a short run equilibrium. Forward looking entry, exit and export decisions determine a more severe selection of incumbents and entrants and an increase in export participation:

$$a_i^{in} = a_A^{in} > a_B^{in} \quad \text{and} \quad a_i^x = a_A^x < a_B^{in} \ .$$

These patterns are suddenly updated at the implementation and then they are fixed for all the future periods $t > i$. Therefore, a trade liberalization causes endogenous job destruction (24) at the time of implementation $\varepsilon_i > 0$, but not later on $\varepsilon_t = 0$ for every $t > i$. This shock takes the labor market (11)–(12) out of its steady state, with higher unemployment and lower job finding probability:

$$u_i > u_A = u_B \quad \text{and} \quad x_i > x_A = x_B \ .$$

The following transitional dynamics of the labor market neither affects the productivity cutoffs nor the export participation. Nevertheless, the adjustment of the industry equilibrium is not immediate. The number of possible entrants financed before the trade liberalization is not sufficient to compensate for the larger exit induced by the policy; which implies $M_i < M_B$. To see this is enough to evaluate the dynamics of the mass of firms (21), which leads to conclude that at the time of implementation the number of firms drops, and even below its long–run level:[20]

$$M_i = \left[1 - T(a_A^{in})\right] \left(\frac{\bar{\pi}_B}{f_e} + \frac{1 - \delta_f}{1 - T(a_B^{in})} \right) M_B < M_A < M_B \ .$$

Given the mass of firms at the time of implementation and the state of the labor market $z_i = \{u_i, x_i\}$, the average wage is determined by the definition of average employment $h\left(\bar{w}_i; z_i\right) f_e/k = (N - u_i)/M_i$. The monotonicity of the hiring rate is sufficient to conclude that the average wage $\bar{w}_i$ exists and is unique, and it can be shown that the average wage at the time of implementation is lower than before the trade liberalization:

$$\bar{w}_i < \bar{w}_B < \bar{w}_A \ .$$

The reason for this result is that the lower job finding probability $x_i < x_B$ decreases the pressure on wages. Two effects should be taken into account. First, the lower job finding probability decreases the value of unemployment during the bargaining. Second, the lower job finding probability decreases the risk that an employed worker receives a wage offer from a better firm. While the first mechanism is common to most search and matching environments, the latter channel is specific to a framework

[20]The analytical derivation of these claims is discussed in Section 7.4 of the appendix.

with OJS and it delivers the key implication that the lower the job finding probability the relative more workers are employed in relatively worse firms.

The intuition that at the time of implementation the relatively worse firms are relatively too big $\frac{h(\bar{w}_i;z_i)}{h_{0i}} < \frac{h(\bar{w}_B;z_B)}{h_{0B}}$ and do not suffer enough poaching from better firms $\frac{\bar{w}_i}{w_{0i}} < \frac{\bar{w}_B}{w_{0B}}$ is also informative about the average profit, through the aggregate equilibrium condition (20). Therefore, the average profit of a domestic firm at the time of implementation is lower than before the trade liberalization:

$$\bar{\pi}_i < \bar{\pi}_B < \bar{\pi}_A \ ,$$

despite the average productivity is higher (because of a left truncation on the previous distribution of productivity). Hence, the reason for this result must be found in the allocation of workers as argued. This completes the determination of the short–run equilibrium, while a more detailed analysis of the impact of trade on welfare is discussed the following paragraph.

4.5 Welfare

In a Melitz type model, trade induced selection implies a higher productivity cutoff and this increases aggregate welfare due to efficiency gains. This argument is also working in the present model, but it is not the only mechanism through which trade leads to welfare gains and losses. In this model unemployment responds to a trade liberalization and firms pay different wages. These two contributions offer novel implications for the effect of a trade liberalization on aggregate welfare.

As in Melitz (2003), the welfare analysis can be based on a firm which is representative of the domestic market in the sense that firm's revenue is equal to average sales per variety. Consider a firm which serves a domestic demand $\bar{q}_t$ making the average revenue per variety $\bar{p}_t\bar{q}_t = \frac{R_t}{(1+\mu_t)M_t} = \frac{\bar{r}_t}{1+\mu_t}$. Substituting in the demand function (1), with $R_t = P_tQ_t$, allows the consumption based price index and the indirect utility from consumption to be written as: $P_t = [(1+\mu_t)M_t]^{-\frac{1}{\sigma-1}}\,\bar{p}_t$ and $Q_t = [(1+\mu_t)M_t]^{\frac{\sigma}{\sigma-1}}\,\bar{q}_t$. It can be shown that the quantity sold in the domestic market is proportional to the demand served by the cutoff firm $q_{0t} = a_t^{in} l_{0t}$, where $l_{0t} = h_{0t}f_e/k$ is the employment of the cutoff firm. Substituting this in the expression for aggregate indirect utility yields welfare:

$$Q_t \;=\; (N-u_t)^{\frac{\sigma}{\sigma-1}}\; l_{0t}^{-\frac{1}{\sigma-1}}\; (\bar{w}_t/w_{0t})^{\frac{\sigma}{\sigma-1}}\; a_t^{in} \tag{25}$$

Aggregate welfare is the outcome of four multiplicative components: total employment $(N-u_t)$, employment at the cutoff firm l_{0t}, the ratio of average over cutoff wage $(\bar{w}_t/w_{0t})$ and the productivity cutoff a_t^{in}. While the latter is the only determinant of welfare in the Melitz's model, in this paper the labor market equilibrium makes the welfare analysis richer. Unemployment $u_t \neq 0$, hiring frictions $h_{0t} \neq 1$ and wage dispersion $\bar{w}_t \neq w_{0t}$ contribute to welfare. Moreover, the trade induced change in the productivity cutoff is suddenly achieved at the time of implementation and it is permanent. Instead, unemployment, job finding probability and the wage ratio evolve over time while the economy follows a transitional dynamics toward the new steady state. Therefore, aggregate welfare follows a transitional dynamics because of the labor market adjustment.

In particular, the definition of aggregate revenue $R_t = P_t Q_t$, the expression for the consumption based price index P_t, the equilibrium condition for the average wage (20) and the wage equation (5) yield $Q_i/Q_B < (R_i/R_B)^{\frac{\sigma-2}{\sigma-1}} (r_{0i}/r_{0B})^{\frac{1}{\sigma-1}}$. Following the discussion in the previous section, it can be shown that the aggregate revenue $R_i < R_B$ is lower at the implementation of a trade liberalization, due to both fewer firms $M_i < M_B$ and lower average revenue $\bar{r}_i < \bar{r}_B$. Moreover, a less tight labor market $x_i < x_B$ implies that more workers are employed at the cutoff firm $h_{0i} > h_{0B}$, even tough it pays lower wage $w_{0i} < w_{0B}$ and makes lower revenue $r_{0i} < r_{0B}$.[21] Therefore, if a trade liberalization occurs in an economy with enough degree of competition, such that the elasticity of substitution is $\sigma \geq 2$, then the aggregate welfare decreases in the short run:

$$Q_i < Q_B < Q_A \ .$$

The sufficient (although not necessary) condition on the elasticity of substitution is mild and very likely to be met, according with the vast majority of empirical studies. More importantly, note that the effect of trade on welfare in the short–run appears to depend not only on aggregate performances but also on the amount of resources allocated to the least productive firms. This intuition is deeply investigated in the following section, which offers a decomposition of the transitional dynamics of welfare.

5 The effect of trade

This framework sheds light on how trade triggers endogenous job destruction in the short–run, but it also shows that trade creates jobs over the transition and does not hurt employment in the long–run. The result that trade–induced firm exit causes unemployment in the short–run is somehow implicit in existing trade models, although the vast majority of them does not address the labor market adjustment. But there is more which can be learned from this model.

Because of OJS, the effect of trade on the labor market goes also through job–to–job reallocations of workers across firms. These flows are driven by employed workers searching for a better wage. Hence, they are complementary to a pure mechanic reallocation of employment which follows the effect of trade on firm market shares; see the discussion in Melitz (2003).[22] At the worker level, trade fosters the search for a better job. At the aggregate level, job–to–job reallocations and their pressure

[21] A detailed discussion of these results is provided in Section 7.4 of the appendix.

[22] Also in this framework, as in other extensions of a Melitz's type model, trade induces a reallocation of output market shares across firms, with only the high productivity firms that gain. This originates an automatic reallocation of labor, which is purely demand driven. In contrast, the novel feature of the present framework is that trade also affects the labor supply curve faced by firms.

on wages become key ingredients of the overall effect of trade on welfare. The model is therefore better suited to address important questions concerning the impact of trade on the labor market: What is the dynamics of the average wage across workers and of the average employment across firms after a trade liberalization? Do all firms adjust employment following the same patterns? How does the labor market contribute to the short–run costs and the long–run gains from trade? Answers to these questions are provided in this section, in which I discuss the implications of the model for the dynamics of employment, wages and welfare in response to an increased trade exposure.

Although the theoretical model is fully characterized analytically (in the previous sections), I illustrate the labor market adjustment and welfare dynamics calibrating the model to the pre–crisis US economy. While I will focus the discussion on the implications of the model (which do not rely on a specific calibration), the numerical exercise will be helpful to visualize the adjustment dynamics. This numerical exercise is merely illustrative, as it neglects several dimensions (such as geography, industry and worker heterogeneity) which are important for a quantitative assessment.[23] However, these features are not necessary to trigger the mechanisms at work in this paper. Also note that following the standard practice on calibrating a Melitz type model is sufficient to deliver predictions which are consistent with the existing quantitative frameworks.

Choice of the parameter values. I feed the model with pre–crisis moments of the US labor market. Between 2000 and 2007 the average unemployment rate in the US is 5.3% and the average unemployment duration is 17.3 weeks.[24] Imposing these moments of the labor market while computing the steady state levels of unemployment and job finding probability yields the implied exogenous separation rate $\delta = 0.18$ and the on–the–job search parameter $\phi = 1.04$.[25] Over the same period, the average number of employees per establishment in manufacturing is 40.28, the average (gross) payroll per worker is $3,818$\$ per month, as reported by the US Census Bureau. Restricting to the sample of firms with 5 to 9 employees the average (gross) payroll per worker is $3,172$\$ per

[23] Among other studies, see Eaton et al. (2011) for the importance of geographic patterns, Dix-Carneiro (2014) for the importance of skill heterogeneity and Helpman et al. (2012) for the importance of a multi–industry setup.

[24] Unemployment rate as a percentage of the population age 16 and older, series LNS14000000 from the Labor Force Statistics of the Current Population Survey. The data on unemployment duration is the series LNS13008275 from the Current Population Survey by the US Bureau of Labor Statistics.

[25] The distribution of unemployment spell duration in weeks is strongly skewed. While the overall average is 17.3 weeks, the median is 9.4 weeks. Given the steady state expressions for unemployment and job finding probability, the on–the–job search parameter is increasing in unemployment duration. Sampling the duration of unemployment spell between the median and the average value yields a value of the on–the–job search parameter lower than 1 but nevertheless substantially larger zero. Moreover, the model would also be consistent with a broader definition of unemployment than the official unemployment rate. For instance, including involuntary part–time workers and discouraged workers the pre–crisis US unemployment rate reaches 10.8%. Computing the on–the–job search parameter with this broader unemployment rate yields $\phi = 0.87$.

month.[26] Under the lens of the model, I impose these two values as the empirical counterparts of the average $\bar{w}$ and the cutoff wage w_0 respectively. Davis et al. (2013) exploit the Job Openings and Labor Turnover Survey and document that, between December 2000 and December 2006, the open positions across establishments in the manufacturing sector are 2.6% of total jobs. Assuming this value as the ratio between vacancies over employment of the representative firm implies a probability of filling a vacancy (in a quarter) for the firm which pays the average wage equal to 0.305. Imposing the value of average employment in the expression for the equilibrium hiring rate evaluated at the average wage yields the size of the initial investment $f_e/k = 132$ relative to the cost of a vacancy. On their studies of US manufacturers, Bernard et al. (2003) and Bernard et al. (2007) estimate an elasticity of substitution $\sigma = 3.79$, and a distribution of productivity which is Pareto with the shape parameter 3.60. I borrow these estimates and I set the location parameter of the productivity distribution to 1. Imposing the average wage and the cutoff wage in the equilibrium expression of the hiring rate yields the size of the fixed production cost relative to the initial investment $f_p/f_e = 1.20$. Given this ratio, setting the expression for the cutoff wage equal to its empirical value yields the cost of posting a vacancy, which amounts to $k = 374\$$ per quarter. I follow Ghironi and Melitz (2005) to fix the exogenous firm destruction rate $\delta_f = 0.025$ and the variable trade cost $\tau = 1.3$. Bernard et al. (2007) document a share of exporting firms of 18%, which I use to determine the implied value of the fixed export cost $f_x = 107\$$ thousands. Finally I normalize the size of the market to $N = 1$. This choice of the parameters allows the initial steady state of the model to be consistent with the salient features of the labor market and export participation of the pre–crisis US economy.

An increase in trade exposure might take the form of a reduction in the variable trade cost τ and/or a reduction in the fixed trade cost f_x. As a reference point for a feasible numerical exercise I consider the results in Dix-Carneiro (2014), which indicate a short–run welfare loss up to 9% and a long–run welfare gain between 2% and 3%. This target is consistent with a continuum of pair values for the percentage change in variable and fixed trade costs $\{d\tau, df_x\} \in (0,1) \times (0,1)$. Without loss of generality for the qualitative analysis that follows, I describe the effect of a trade liberalization which suddenly decreases variable trade costs by $d\tau = -5\%$ and fixed trade costs by $df_x = -25\%$.

5.1 Adjustment of the labor market

The system (11)–(12) shows how a trade–induced endogenous job destruction ε_{t+1} moves the labor market allocation to an unstable short–run equilibrium. The adjustment of the labor market is driven by the autonomous dynamics of the unemployment u_t. The transmission channel (20)–(21) links average wage and average employment with average profit and mass of incumbent firms, thus

[26] The 2007 Annual Survey of Manufactures reports 332,536 establishments, with a value added of 2,382,643,001 thousands \$, an annual payroll of 613,768,568 thousands \$ and 13,395,670 employees. In the subset of firms with 5 to 9 employees the annual payroll is 12,601,162 thousands \$ and the employees are 331,043.

it triggers the dynamic adjustment of the industry equilibrium.

Figure (2) shows the transitional dynamics of unemployment u_t, job finding probability x_t, average real wage $\bar{w}_t/P_t$ across workers and average employment across firms $(N - u_t)/M_t$; where the latter two variables are relative to their steady state levels before the trade liberalization. The policy occurs at the beginning of the second quarter and the horizon of the numerical exercise shows up to ten years after the policy implementation on a quarterly basis. At the impact, trade–induced

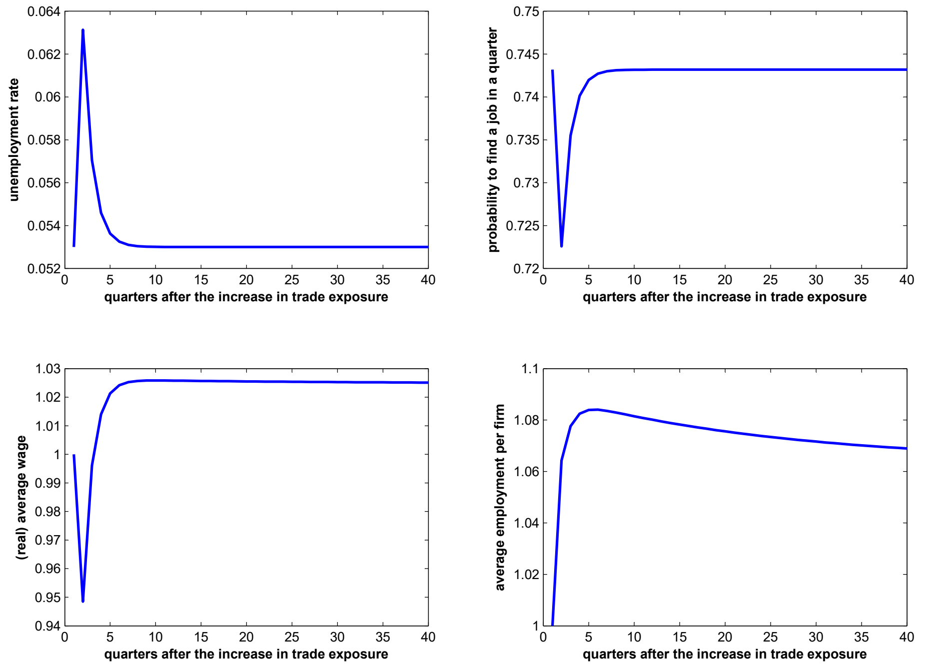

Figure 2: Adjustment of the labor market.

firm exit causes a sudden increase of unemployment, from 5.3% to about 6.3%. The expected duration of an unemployment spell (the inverse of the job finding probability) increases from 17.3 weeks to about 18 weeks. Wages respond, both directly through $\lambda(u_t)$ which shows that current value of a job in the labor market falls; and indirectly, since the job finding probability falls, the value of searching and the outside option of unemployed workers decrease. The reduction in the price index mitigates the loss in real wages, but still the average real wage decreases at the impact by more than 5%. In the following periods, the autonomous dynamics of unemployment converges toward its steady state, and doing so it triggers the increase in job finding probability, hence generating upward pressure on wages.

The general equilibrium linkages between labor market and output market also play a role, and this can be inferred by looking at the dynamics of average employment. When trade induces the exit of low productivity (hence small) firms, the average employment increases by more than 6%.

Within this jump there is of course a mechanic contribution due to the selection of firms, but this is neither the entire explanation, nor the more interesting insight. In fact, in the following periods the average employment continues to increase and overshoots what will be the long–run level, reaching a change greater than 8%. The reason for this pattern is that after the trade liberalization the labor market is less tight (x_t falls), thus all firms have now a higher probability to fill their vacancies, which were posted in the previous period. Ceteris paribus, the weaker competition in the labor market increases employment at every incumbent firm (even tough the change is not symmetric as I will discuss later). As a consequence, in the short–run the average employment is too large. Together with an excessively high unemployment, this result implies that there are too few firms with respect to the long–run equilibrium. These effects of a weaker competition in the labor market do not die away for the entire transition, that is as long as the labor market remains less tight than in steady state.

5.2 Worker reallocation across firms

After the implementation of a trade liberalization the unemployment is too high and the job finding probability is too low; as a consequence, firms that remain in the market after a trade liberalization benefit from an excessively low labor market tightness. Employment in the highest productivity firms is proportional to $h(w_{1t}; z_t) = (1 - x_t)$, while employment in the lowest productivity firms is proportional to $h(w_{0t}; z_t) = (1 - x_t)(1 - \phi x_t)$. Therefore, both subsets of firms employ more in the short–run than they will in the long run, and in relative terms the excess of employment in the least productive firms is higher. As a consequence, over the transition (while x_t converges to its steady state) the reduction in employment hits the low productivity firms more. The result is a progressively higher concentration of workers employed at relatively high productivity firms. This is the mechanism leading to a higher average wage in the long–run with respect to before the trade liberalization; as discussed in Section 4.

Figure (3) shows the evolution of employment relative to the initial steady state for three types of firms: a firm endowed with the cutoff productivity after the trade liberalization (*bold blue line*); a more productive firm which still does not export (*dashed blue line*); a firm which becomes exporter after the trade liberalization (*dashed red line*). In the current model, thanks to OJS, the employment of a firm is determined by the wage the firm pays relative to the distribution of wages paid by all other firms. This is the reason why, the response of employment at the time of policy implementation differs among the firm types. The firm which is endowed with the new productivity cutoff $a_A^{in} > a_B^{in}$ is the firm which pays the lowest wage among incumbents; this was not the case before the trade liberalization. Therefore, the cutoff firm reduces the wage and doing so reduces employment, by more than 20% at the impact. In the following periods the cutoff firm reduces employment even

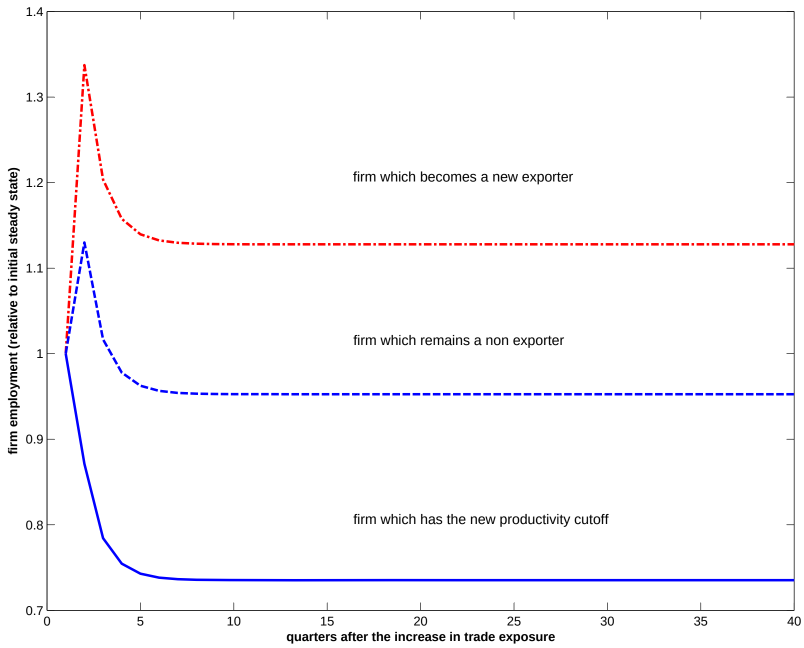

Figure 3: Firm employment, by type of firm.

further as the labor market becomes more tight; and this is common to all firms, as previously discussed.

Firms with higher productivity (whether they become exporters or not) respond to the trade liberalization with an increase of employment. This is due to the fact that these firms do not suffer a worsening of their position in the wage distribution, while they fill more vacancies due to a lower labor market tightness. However, only the firms which becomes a new exporter increase employment permanently. Firms that do not become exporters might experience a temporary increase of employment, if their productivity is not too low. But in the long–run they will contract the number of employees below the level they had before the trade liberalization.

5.3 Welfare

The expression for aggregate welfare (25) can be easily handled to deliver predictions for welfare per capita Q_t/N and welfare per employed worker $Q_t/(N - u_t)$, or even to obtain a measure of welfare which does not depend on market size $Q_t/N^{\frac{\sigma}{\sigma-1}}$ but only on the unemployment rate u_t/N. In all these transformations, welfare decreases with the unemployment rate, decreases the larger is the employment at the cutoff firm, increases the higher is the wage ratio and it is greater the higher the cutoff productivity.

Figure (4) describes the short–run dynamics of welfare up to three years after a trade liberaliza-

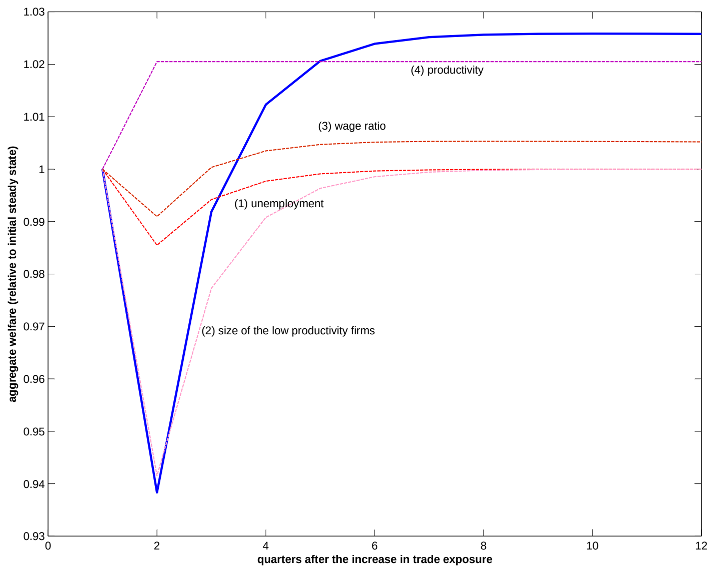

Figure 4: Aggregate welfare, and its 4 components.

tion, which occurs at the beginning of the second quarter. The *bold blue line* is welfare normalized by the exogenous market size $Q_t/N^{\frac{\sigma}{\sigma-1}}$; the *dashed lines* are the values of the four components in (25), that is: (1) unemployment rate u_t/N, (2) employment at the cutoff firm $l_{0t}^{-\frac{1}{\sigma-1}}$, (3) wage ratio $(\bar{w}_t/w_{0t})^{\frac{\sigma}{\sigma-1}}$, (4) cutoff productivity a_t^{in}. All values are relative to the corresponding levels before the trade liberalization and by construction the product of the four contributions yields welfare at each point in time. Thus a contribution lower (higher) than 1 decreases (increases) welfare. The short–run welfare costs are due to the labor market adjustment, while the trade induced productivity gains are capitalized suddenly after the trade liberalization and are permanent.

The model is suitable for a welfare decomposition which sheds light on the main channels of costs and gains from trade over time. The main cause of short–run costs from trade is the excessive size of low productivity firms. The excessively low competition in the labor market protects the low productivity firms and slows down the reallocation of workers toward more productive firms. This process alone is responsible for almost 6% of welfare costs and it substantially erodes welfare for about two years. The spike in unemployment after trade liberalization is the second source of welfare costs. In isolation, this channel causes 1.5% loss of welfare and contributes to weaken welfare for about one year. Notice that the negative contributions of the size of low productivity firms and unemployment vanish in the long run, as the labor market tightness approaches its unique steady state value (which is the same before and after the trade liberalization).

The labor market adjustment contributes to the effect of trade on welfare also through the change in the average wage $\bar{w}_t$ relative to the lower bound of the wage support w_{0t}. Suddenly after the trade liberalization the wage ratio is lower.[27] In the following periods, as the labor market becomes more tight both wages increase, but the average wage increase is stronger due to the change in the composition of the workforce. The concentration of workers employed in relatively high productivity firms increases and this yields a higher average wage $\bar{w}_t$ than before the trade liberalization; instead, the low wage w_{0t} approaches the same steady state level as before the trade liberalization from below. Thus, after a temporary reduction, the increase in the wage ratio becomes permanent and provides a contribution of about 0.5% to the long–run welfare gains from trade.

The previous analysis shows that although the labor market adjustment after a trade liberalization destroys welfare in the short–run, it also generates additional welfare gains in the long–run which are neither captured nor depend on the classical trade–induced increase in the cutoff productivity. These gains arise because of job–to–job reallocations due to employed workers searching for higher wages. In terms of magnitude, the increase in average wage provides a contribution as large as one fourth of the one by productivity, about one fifth of the total long–run gains from trade.

6 Conclusion

This paper has developed a theoretical framework which explains the effect of trade on unemployment, worker reallocation, wages and welfare, describing the transitional dynamics of the labor market adjustment in response to increased trade exposure. The paper shows how frictions behind the labor market adjustment affect how trade impacts efficiency and welfare over time.

Although in the long–run welfare gains from trade are unambiguous, in the short–run welfare costs are predominant. The response of a frictional labor market to a trade–induced firm exit is characterized by a rise in unemployment, a reduced probability of finding a job, a cut in wages, and a less efficient allocation of workers across firms. Nevertheless, the model also shows that this short–run "bad" equilibrium outcome is not stable. Trade–induced selection of firms triggers the reallocation of workers from relatively low productivity firms to more productive firms which become exporters. During the adjustment, the probability of finding a job grows, and under this pressure, the wages increase and relatively low productivity firms shrink. In the long–run, the model shows welfare gains from trade, not only because of a greater aggregate productivity, but also because the average wage is higher and more workers are employed in relatively more productive firms.

While the model is consistent with fears and hopes concerning international trade, both the predictions and the mechanisms of the model find support in the recent empirical work. Clearly these patterns can only be studied in a dynamic framework, because a simple steady state comparison hides the short–run costs due to the labor market adjustment. Moreover, accounting for the observed inter–firm reallocation requires a channel through which firms that become exporters have the incentive to

[27] See the discussion on the impact of trade in the appendix for the analytical derivation of this result.

pay better wages and grow also by poaching workers from other firms. Toward this goal, the model features a frictional labor market with wage bargaining and OJS, along the lines of Mortensen (2010). Importantly, the present model extends the original steady state treatment of the labor market to a dynamic setup, which also accounts for firm entry and endogenous job destruction. Combining this structure of the labor market with a Melitz (2003) output market yields a dynamic general equilibrium with wage dispersion in which more productive firms pay better wage, employ more workers and select into the export market.

The model remains highly tractable, also in the derivation of the transitional dynamics. This is achieved by means of an analytical closed form solution for the law of motion of unemployment and of the probability to find a job. Given this information, hirings and separations at each point of the wage distribution are determined. This shows what can be learned through the present model. At the micro–level, the model predicts the patterns of inter–firm reallocation of workers, and the change in wages which affect workers depending on their position in the wage distribution. At the aggregate level, the model predicts the dynamics of unemployment, average wage and average employment which follow a trade–induced labor market adjustment. Although firm entry, exit and export decisions can be thought to be forward looking and correctly anticipated, aggregate productivity and indeed welfare follow a transitional dynamics induced by the labor market adjustment.

It is therefore important to have a model which can be used to design policies that balance the short–run costs and the long–run gains from international trade. Based on what can be learned from this model, the time that the economy spends in a costly transition depends on the functioning of the domestic labor market. Therefore, there is room for labor market policies that reduce the time of adjustment while not hindering the beneficial trade–induced reallocation process.

7 Appendix

7.1 Labor market equilibrium

In this section I present the matching technology which is consistent with the labor market equilibrium discussed in the paper and the Bellman equations for the value of employment and unemployment which prescribe the worker policies. In the second part of this paragraph, I discuss the solution of the Bernoulli equation which yields the equilibrium hiring rate $h(w; z_t)$. Finally I analyze the dynamics of unemployment and job finding probability.

Matching technology. Each unemployed worker sent one application in the previous period, whereas each employed worker sent $\phi > 0$ applications. Firms posted $\mathcal{V}_{t-1}$ vacancies, but because of exogenous and endogenous exit only a fraction ρ_t of these is still open at time t. Therefore, every period t, after firm entry and exit occur, a mass of $\rho_t \mathcal{V}_{t-1}$ vacancies is matched with $u_{t-1} + \phi n_{t-1}$ job applications. Matches are formed according to a technology which combines current vacancies $\rho_t \mathcal{V}_{t-1}$ and job applications $u_{t-1} + \phi n_{t-1}$. The number of matches is given by:

$$\mathcal{M}_t = \frac{\rho_t \mathcal{V}_{t-1} \left(u_{t-1} + \phi n_{t-1}\right)}{\rho_t \mathcal{V}_{t-1} + \left(u_{t-1} + \phi n_{t-1}\right)}$$

This functional form of matching technology is a special case of the one proposed by Ramey et al. (2000) and has the advantage of a matching probability bounded in the unit interval for both workers and firms, while the technology is homogeneous of degree 1 for vacancies and job applications, as documented in the empirical literature. Labor market tightness θ_t and the probability that a worker receives a job offer x_t are:

$$\theta_t = \frac{\rho_t \mathcal{V}_{t-1}}{u_{t-1} + \phi n_{t-1}} \qquad\qquad x_t = \left(1 + 1/\theta_t\right)^{-1}$$

while the probability that a vacancy is visited by a worker is: $x_t/\theta_t = 1 - x_t$.

Value of unemployment and value of employment. Let $U\left(z_t\right)$ be the value of being unemployed and $W\left(a; z_t\right)$ be the value of being employed in a firm with productivity a when the state of the labor market is z_t. The distribution of wage offers over the productivity support $a \geq a_t^{in}$ is $\mathcal{F}(a; z_t) = F(\omega(a; z_t); z_t)$, as implied by the wage–productivity assignment (17). Conditional on receiving a wage offer in the next period, the expected value from searching for a worker who starts period t employed in a firm with productivity a is $S\left(a; z_{t+1}\right)$. Workers matched with employers endowed with productivity a accept the wage offer of a firm endowed with productivity a' if and only if $W\left(a'; z_{t+1}\right) \geq W\left(a; z_{t+1}\right)$. The value from searching is:

$$\begin{aligned} S\left(a; z_{t+1}\right) &= \int_{a_{t+1}^{in}}^{\infty} \max\left\{W\left(a'; z_{t+1}\right), W\left(a; z_{t+1}\right)\right\} d\mathcal{F}(a; z_t) \\ &= W\left(a; z_{t+1}\right) + \mathcal{G}\left(a; z_{t+1}\right) \end{aligned}$$

where

$$\mathcal{G}\left(a;z_{t+1}\right)=\left[1-\mathcal{F}(a;z_{t+1})\right]\int_{a}^{\infty}\left[W\left(a';z_{t+1}\right)-W\left(a;z_{t+1}\right)\right]\frac{d\mathcal{F}(a';z_{t+1})}{1-\mathcal{F}(a;z_{t+1})}$$

is the (ex–ante) expected gain of voluntary quitting the job at a firm with productivity a for a firm with productivity higher or equal $m(a;z_t)$. Notice that $\mathcal{G}\left(a;z_{t+1}\right)\geq 0$ and the better the wage paid by the current employer the lower the margins to improve $\frac{d\mathcal{G}(a;z_{t+1})}{da}\leq 0$. Furthermore, notice that the value from searching for a worker who is employed at the marginal producer $S\left(a_{t+1}^{in};z_{t+1}\right)$ is equal to the average value of being employed.

Unemployed workers do not earn labor income in the current period, they send one job application and they receive a job offer in the following period with probability x_{t+1}. The value of searching for an unemployed worker (who does not have the option to remain matched with a current employer) is equal to the value of searching starting from the marginal producer $S\left(a_{t+1}^{in};z_{t+1}\right)$. The value of being unemployed at time t is:

$$U\left(z_t\right)=x_{t+1}S\left(a_{t+1}^{in};z_{t+1}\right)+\left(1-x_{t+1}\right)U\left(z_{t+1}\right)$$

Employed workers in a firm endowed with productivity a earn a wage $\omega\left(a;z_t\right)$, send $\psi\in[0,\phi]$ job applications and suffer a loss of value $\psi\epsilon$ if they search; where the parameter $\epsilon\geq 0$ indicates the effort cost of applying for a job when employed. In the next period, employed workers might become unemployed because of an exogenous shock hits the firm or the job, with arrival rate δ. Otherwise the worker receives a renewal of the contract with the current employer. Regardless of the new employment status, an employed worker who sends ψ applications in the current period receives a wage offer in the future period with probability ψx_{t+1}. The value of employment is:

$$\begin{aligned}W\left(a;z_t\right) &= \omega\left(a;z_t\right)-\psi\epsilon+\\ &+ \left(1-\delta\right)\left[\left(1-\psi x_{t+1}\right)W\left(a;z_{t+1}\right)+\psi x_{t+1}S\left(a;z_{t+1}\right)\right]\\ &+ \delta\left[\left(1-\psi x_{t+1}\right)U\left(z_{t+1}\right)+\psi x_{t+1}S\left(a_{t+1}^{in};z_{t+1}\right)\right]\end{aligned}$$

where $\omega\left(a;z_t\right)$ is the continuous function mapping productivity into wage and, as discussed in the paper, the wage is a strictly positive increasing function of productivity. Since the gains from a voluntary reallocation are not increasing in the productivity of the current employer, the value of employment $W\left(a;z_t\right)$ is increasing in the productivity of the current employer a.

Worker policy. Comparing the value of being employed and sending ψ applications with the value of being employed and sending zero applications yields the benefit from searching $\psi x_{t+1}\left[(1-\delta)\left(S\left(a;z_{t+1}\right)-W\left(a;z_{t+1}\right)\right)+\delta\left(S\left(a_{t+1}^{in};z_{t+1}\right)-U\left(z_{t+1}\right)\right)\right]$ which is a decreasing function of the productivity of the current employer. The cost of searching which is constant $\psi\epsilon$. Therefore, if employees of firms at the top of the productivity distribution search then all employed workers search. The benefit of searching for $a \to \infty$ converges to $\delta\psi x_{t+1}\left[S\left(a_{t+1}^{in}; z_{t+1}\right) - U\left(z_{t+1}\right)\right]$; that is the expected value of avoiding unemployment in case the current job is destroyed. I discuss the equilibrium of the labor market when workers with the best employment status are indifferent between searching or not $\delta x_{t+1}\left[S\left(a_{t+1}^{in}; z_{t+1}\right) - U\left(z_{t+1}\right)\right] = \epsilon$ and I assume they search.[28] Therefore all employed workers search, and they send the maximum number of applications $\psi = \phi > 0$, regardless the effort cost. Substituting this indifference condition in the value of employment and unemployment yields the surplus of being employed:

$$\begin{aligned} W(a; z_t) - U(z_t) &= \omega(a; z_t) - \epsilon/\delta + (1-\delta)\left[W(a; z_{t+1}) - U(z_{t+1})\right] + \\ &+ \quad (1-\delta)\,\phi x_{t+1}\left[S(a; z_{t+1}) - W(a; z_{t+1})\right] \end{aligned}$$

where $\omega(a; z_t) > \epsilon/\delta$ for every t is a sufficient condition for $W(a; z_t) > U(z_t)$. A reservation wage w_R exists such that $W(a; z_t) = U(z_t)$ for $\omega(a; z_t) = w_R$. Let $S_R(z_{t+1}) \geq S\left(a_{t+1}^{in}; z_{t+1}\right)$ and $W_R(z_{t+1}) \geq U(z_{t+1})$ be the value of searching and being employed starting from an employment at the reservation wage; then it can be shown by substitution that the reservation wage is decreasing in both $S_R(z_{t+1})$ and $W_R(z_{t+1})$. Hence, the maximum reservation wage $w_R(z_t)$ is obtained when $W_R(z_{t+1}) = U_{t+1}$ and $S_R(z_{t+1}) = S\left(a_{t+1}^{in}; z_{t+1}\right)$, which yields:

$$\begin{aligned} w_R &= \epsilon/\delta - (1-\delta)\,\phi x_{t+1}\left[S\left(a_{t+1}^{in}; z_{t+1}\right) - U\left(z_{t+1}\right)\right] \\ &= [1 - \phi(1-\delta)]\epsilon/\delta \end{aligned}$$

Notice that the maximum reservation wage is constant over time, equal to the effort cost when employed workers send as many applications as unemployed workers $\phi = 1$, lower (greater) than the effort cost if employed workers search more (less) than unemployed workers.[29] The policy of unemployed workers is established: for a lower bound of the wage distribution $w_{0t} > w_R$ unemployed workers accept every wage offer.

Derivation of the equilibrium hiring rate. The wage equation (5) implies $\frac{\partial r(a,l_t)}{\partial l_t} - w(a, l_t) - \frac{\partial w(a,l_t)}{\partial l_t} l_t = w(a, l_t)$. The profit of a firm endowed with productivity a which employs l_t workers is $\pi(a, l_t; z_t) = r(a, l_t) - w(a, l_t)l_t - k\vartheta(a, l_t; z_t) - f_p - \mathbf{1}(a)f_x$. If firms do not have the incentive to deviate from the employment level implied by wage (5) then the condition $\frac{\partial \pi(a,l_t;z_t)}{\partial l_t} = 0$ holds and it implies:

$$\frac{\partial \vartheta(a, l_t; z_t)}{\partial l_t} = \frac{1}{k}\left(\frac{\partial r(a, l_t; \zeta_t)}{\partial l_t} - \frac{\partial w(a, l_t; \zeta_t)}{\partial l_t} l_t - w(a, l_t)\right) = \frac{w(a, l_t)}{k}$$

[28] Closing the worker policy through a corner solution has the advantage of simplifying the equilibrium of the labor market, while preserving the main contribution of OJS. Alternatively, Cahuc et al. (2006) provide a labor model with OJS and wage bargaining in which the searching effort is endogenous to the current wage. They show how that model is suitable to rationalize the salient observed patterns of the wage distribution.

[29] This feature is common to models with OJS, it reflects the different probability of finding a job when starting in the two employment status.

The partial derivative of the policy for vacancies (15) with respect to employment yields

$$\begin{aligned}\frac{\partial \vartheta(a,l_t;z_t)}{\partial l_t} &= \frac{h(w(a,l_t);z_t) - \frac{h(w(a,l_t);z_t)}{\partial w}\frac{\partial w(a,l_t)}{\partial l_t} l_t}{h(w(a,l_t);z_t)^2} - \lambda(u_t) \\ &= \frac{h(w(a,l_t);z_t) + \frac{1}{\sigma}\frac{h(w(a,l_t);z_t)}{\partial w} w}{h(w(a,l_t);z_t)^2} - \lambda(u_t)\end{aligned}$$

where the second equality is obtained after the substitution $\frac{\partial w(a,l_t)}{\partial l_t} l_t = -\frac{w}{\sigma}$ as implied by the wage equation (5). The system of the two conditions yields:

$$\frac{h(w;z_t)}{\partial w} + \frac{\sigma}{w} h(w;z_t) = \left(\frac{\sigma}{k} + \frac{\sigma\lambda(u_t)}{w}\right) h(w;z_t)^2$$

which is the canonical form of a Bernoulli differential equation. The general solution is:

$$h(w;z_t) = \frac{1}{\frac{\sigma}{\sigma-1}\frac{1}{k} w + \lambda(u_t) + w^{\sigma} c}$$

where c is the constant of integration. The boundary condition $h(w_{0t};z_t) = h_{0t}$ yields

$$c = \left(1 - \frac{\sigma}{\sigma-1}\frac{h_{0t}}{k} w_{0t} - h_{0t}\lambda(u_t)\right)\frac{1}{h_{0t} w_{0t}^{\sigma}}$$

hence the unique particular solution passing through the boundary condition is

$$h(w;z_t) = \frac{h_{0t}}{h_{0t}\lambda(u_t) + \frac{\sigma}{\sigma-1}\frac{w_{0t}h_{0t}}{k}\frac{w}{w_{0t}} - \left(h_{0t}\lambda(u_t) + \frac{\sigma}{\sigma-1}\frac{w_{0t}h_{0t}}{k}\frac{w}{w_{0t}} - 1\right)\left(\frac{w}{w_{0t}}\right)^{\sigma}}$$

which is (16) for $\alpha(z_t) = \lambda(u_t) h_{0t}$, $\beta(z_t) = \frac{\sigma}{\sigma-1}\frac{w_{0t}h_{0t}}{k}$.

Analysis of the labor market dynamics. Substituting for $1 - x_{t+1} = \frac{u_t + (\delta+\varepsilon_{t+1})(N-u_t)}{u_t+\phi(N-u_t)} - (1-\phi)\frac{N-u_t}{u_t+\phi(N-u_t)}$ and $1 - \phi x_{t+1} = \frac{u_t+(\delta+\varepsilon_{t+1})(N-u_t)}{u_t+\phi(N-u_t)}$ from (12) in (11) yields a quadratic equation in u_t. The system of difference equations which governs the dynamics of the labor market is:

$$\begin{aligned} u_{t+1} &= \frac{[u_t + (\delta+\varepsilon_{t+1})(N-u_t)]^2}{u_t+\phi(N-u_t)} - \frac{(1-\phi)(N-u_t)u_t}{u_t+\phi(N-u_t)} \\ x_{t+1} &= \frac{(1-\delta)(N-u_t)}{u_t+\phi(N-u_t)}\left(1 - \frac{\varepsilon_{t+1}}{1-\delta}\right)\end{aligned}$$

Notice that for $u_t \to N$ then $u_{t+1} = N$ and for $u_t \to 0$ then $u_{t+1} = \frac{(\delta+\varepsilon_{t+1})^2}{\phi} N > 0$. Let $A = \delta + \varepsilon_{t+1} \in (0,1)$, $B = [(1-A)^2 + (1-\phi)]$ and $D = [2A(1-A) - (1-\phi)]$ then

$$\begin{aligned} u_{t+1} &= \frac{B u_t^2 + D N u_t + A^2 N^2}{(1-\phi)u_t + \phi N} \\ \frac{du_{t+1}}{du_t} &= \frac{B(1-\phi)u_t^2 + B2\phi N u_t + \left(\phi D - (1-\phi)A^2\right) N^2}{[(1-\phi)u_t + \phi N]^2}\end{aligned}$$

Notice that the minimum level of unemployment implied by the law of motion is $\phi u_t = A^2 N$. Substituting implies that for every $\phi \in [\delta + \varepsilon_{t+1}, 1]$ if $2B > 1-\phi$ then the numerator of the first order derivative is strictly positive. This sufficient condition is always satisfied as $B > 1-\phi$, therefore

$\frac{du_{t+1}}{du_t} > 0$. In order to determine the convexity I organize the discussion in two cases. For $\phi = 1$ it is immediate to show that the second order derivative is positive. For $\phi < 1$ it is convenient to notice that the derivative of the numerator with respect to u_t is $B/(1-\phi)$ times the derivative of the denominator with respect to u_t; and the denominator is increasing in u_t. Therefore

$$\begin{aligned}\frac{d^2u_{t+1}}{d^2u_t} > 0 \quad &\Longleftrightarrow \quad \frac{B}{1-\phi}\left[(1-\phi)u_t + \phi N\right]^2 > B(1-\phi)u_t^2 + B2\phi N u_t + \left(\phi D - (1-\phi)A^2\right)N^2 \\ &\Longleftrightarrow \quad B\phi^2 + A^2(1-\phi)^2 > (1-\phi)\phi D \\ &\Longleftrightarrow \quad \frac{(1-A)^2\phi}{1-\phi} + \frac{A^2(1-\phi)}{\phi} + 1 > 2A(1-A)\end{aligned}$$

which is always satisfied since $A = \delta + \varepsilon_{t+1} \in (0,1)$. This concludes the proof. The dynamic equation for u_{t+1} is a positive, increasing and convex function of u_t. Since u_{t+1} attains a strictly positive value as $u_t \to 0$ the 45–degree line cuts from below, which implies the existence and uniqueness of one steady state and a stable trajectory.

7.2 Industry equilibrium

In this section I discuss the derivation of the profit function and the revenue function. Furthermore, I prove existence and uniqueness of the productivity cutoffs a_t^{in} and a_t^x and I prove the impact of a trade liberalization on the industry equilibrium: higher productivity cutoff $\frac{da_t^{in}}{df_x} < 0$, lower export cutoff $\frac{da_t^{in}}{df_x} > 0$ and higher average profit plus average fixed costs $\frac{d(\bar{\pi}_t + \bar{f}_t)}{df_x} < 0$.

Derivation of the profit function. Rearranging the expression for the hiring rate (16) to explicit for the ratio $\frac{h(w;z_t)}{h_{0t}}\left(\frac{w}{w_{0t}}\right)^\sigma$ then substituting for $\frac{h(w;z_t)}{h_{0t}}\left(\frac{w}{w_{0t}}\right)^\sigma = \left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1}$ from (17) yields

$$\left(\frac{\alpha(z_t)}{h_{0t}} + \frac{\beta(z_t)}{w_{0t}h_{0t}}w\right)h\left(w;z_t\right) = \left(1 + \left(1+\mathbf{1}(a)\tau^{1-\sigma}\right)\left(\alpha(z_t)+\beta(z_t)-1\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1}\right).$$

Employment satisfies $l_t = h\left(w;z_t\right)\frac{f_e}{k}$, which substituting in the previous expression allows labor payroll to be written

$$wl_t = \frac{w_{0t}h_{0t}}{\beta(z_t)k}\left[\left(1 + \left(1+\mathbf{1}(a)\tau^{1-\sigma}\right)\left(\alpha(z_t)+\beta(z_t)-1\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1}\right)f_e - \frac{\alpha(z_t)k}{h_{0t}}l_t\right]$$

Looking at the the profit function $\pi(a,l_t;z_t) = \frac{\sigma}{\sigma-1}w(a,l_t)l_t + \lambda(u_t)kl_t - f_e - f_p - \mathbf{1}(a,l_t)f_x$ and substituting for the vacancy policy (15) it is then immediate to substitute back for $\alpha(z_t) = \lambda(u_t)h_{0t}$, $\beta(z_t) = \frac{\sigma}{\sigma-1}\frac{w_{0t}h_{0t}}{k}$ where it is convenient, such that labor payroll can be written as follows:

$$wl_t = \frac{\sigma-1}{\sigma}\left[\left(1+\mathbf{1}(a)\tau^{1-\sigma}\right)f_e\left(\alpha(z_t)+\beta(z_t)-1\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1} + f_e - \lambda(u_t)kl_t\right]$$

Substituting in (**??**) yields the profit:

$$\pi(a;z_t) = \left(1+\mathbf{1}(a)\tau^{1-\sigma}\right)f_e\left(\alpha(z_t)+\beta(z_t)-1\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1} - f_p - \mathbf{1}(a)f_x\ .$$

Derivation of the revenue function. The condition $\pi(a_t^{in}; a_t^{in}) = 0$ implies $f_e\left(\alpha(z_t) + \beta(z_t) - 1\right) = f_p$, which substituting in the expression for labor payroll yields:

$$wl_t = \frac{\sigma - 1}{\sigma}\left[\left(1 + \mathbf{1}(a)\tau^{1-\sigma}\right)\left(\frac{a}{a_t^{in}}\right)^{\sigma-1} f_p + f_e - \lambda(u_t)kl_t\right]$$

Recognizing the same functional form of the profit $\pi(a; a_t^{in})$ yields:

$$wl_t = \frac{\sigma - 1}{\sigma}\left[\pi(a; a_t^{in}) + f_p + \mathbf{1}(a)f_x + f_e - \lambda(u_t)kl_t\right]$$

The wage equation (5) can now be used to explicit the revenue as a function of profit and employment:

$$r(a, l_t; z_t) = \frac{2\sigma - 1}{\sigma}\left[\pi(a; a_t^{in}) + f_p + \mathbf{1}(a)f_x + f_e - \lambda(u_t)kl_t\right]$$

The average revenue across firms is:

$$\bar{r}_t = \frac{2\sigma - 1}{\sigma}\left[\bar{\pi}_t + f_p + \mu_t f_x + f_e - \lambda(u_t)k\frac{(N - u_t)}{M_t}\right]$$

where $(N - u_t)/M_t$ is the average employment per firm by definition.

Existence and uniqueness of the productivity cutoffs, and the impact of trade. As claimed in the text, the proof of existence and uniqueness of a cutoff productivity a^{in} is equivalent to the one in Melitz (2003). Let $\kappa(a') = \left(\frac{\varphi(a')}{a'}\right)^{\sigma-1} - 1 = \int_{a'}^{\infty}\left(\frac{a}{a'}\right)^{\sigma-1}\frac{dT(a)}{1-T(a')} - 1 > 0$. An application of the Leibniz rule yields:

$$\frac{d\kappa(a')}{da'} = \frac{\kappa(a')dT(a')/da'}{1 - T(a')} - (\sigma - 1)\frac{1 + \kappa(a')}{a'}$$

Define the function

$$j(a') = [1 - T(a')]\kappa(a') \geq 0$$

then the following properties hold:

$$\frac{dj(a')}{da'} = -\frac{1}{a'}(\sigma - 1)[1 - T(a')][1 + \kappa(a')] < 0 \quad \text{and} \quad \frac{dj(a')}{da'}\frac{a'}{j(a')} = -(\sigma - 1)\frac{1 + \kappa(a')}{\kappa(a')} < -(\sigma - 1)\,.$$

The free entry condition (23) can be written as:

$$j(a_t^{in})f_p + j(a_t^x)f_x = \delta_f f_e$$

where $a_t^x = \tau(f_x/f_p)^{\frac{1}{\sigma-1}}a_t^{in}$. Hence the left hand side is monotonically decreasing in a_t^{in} on the positive real line $(0, \infty)$. The right hand side is constant and strictly positive, which implies existence and uniqueness of a_t^{in}. In open economy, keeping a_t^{in} constant a change in f_x implies a change in $j(a_t^x)$ as follows:

$$\frac{\partial j(a_t^x)}{\partial a_t^x}\frac{\partial a_t^x}{\partial f_x}f_x + j(a_t^x) = \left(\frac{\partial j(a_t^x)}{\partial a_t^x}\frac{a_t^x}{j(a_t^x)}\frac{\partial a_t^x}{\partial f_x}\frac{f_x}{a_t^x} + 1\right)j(a_t^x) < 0$$

where the sign is implied by $\frac{\partial a_t^x}{\partial f_x}\frac{f_x}{a_t^x} = \frac{1}{\sigma-1}$ and the elasticity of the $j(a')$ function. The right hand side of the free entry condition is constant, therefore the new intersection identifies a new productivity cutoff $\frac{da_t^{in}}{df_x} < 0$. The change in a_t^x is $\frac{da_t^x}{df_x} = \frac{a_t^x}{a_t^{in}}\frac{da_t^{in}}{df_x} + \frac{1}{\sigma-1}\frac{a_t^x}{f_x}$, therefore an evaluation of $\frac{da_t^{in}}{df_x}$ is necessary to determine the sign of $\frac{da_t^x}{df_x}$. The average profit can be written as:

$$\bar{\pi}_t = \kappa(a_t^{in})f_p + \frac{1-T(a_t^x)}{1-T(a_t^{in})}\kappa(a_t^x)f_x$$

Rewriting the free entry condition and taking the total derivative with respect to f_x yields:

$$\begin{aligned}
\frac{dj(a_t^{in})}{da_t^{in}}\frac{da_t^{in}}{df_x}f_p &= -\frac{dj(a_t^x)}{da_t^x}\frac{da_t^x}{df_x}f_x - j(a_t^x) \\
\frac{dj(a_t^{in})}{da_t^{in}}\frac{da_t^{in}}{df_x}f_p &= -\frac{dj(a_t^x)}{da_t^x}\left(\frac{a_t^x}{a_t^{in}}\frac{da_t^{in}}{df_x} + \frac{1}{\sigma-1}\frac{a_t^x}{f_x}\right)f_x - j(a_t^x) \\
\frac{da_t^{in}}{df_x} &= \frac{1-T(a_t^x)}{\frac{dj(a_t^{in})}{da_t^{in}}f_p + \frac{dj(a_t^x)}{da_t^x}\frac{a_t^x}{a_t^{in}}f_x} < 0
\end{aligned}$$

where in the second line the change in a_t^x is implied by the export indifference condition: $\frac{da_t^x}{df_x} = \frac{a_t^x}{a_t^{in}}\frac{da_t^{in}}{df_x} + \frac{1}{\sigma-1}\frac{a_t^x}{f_x}$ and the sign of the third line is implied by $dj(a')/da' < 0$. Rewriting the first line yields

$$\frac{da_t^x}{df_x} = \frac{-1}{\frac{dj(a_t^x)}{da_t^x}f_x}\left(\frac{dj(a_t^{in})}{da_t^{in}}\frac{da_t^{in}}{df_x}f_p + j(a_t^x)\right) > 0$$

where the expression in the brackets is positive, hence the sign is determined.

The free entry condition allows $\bar{\pi}_t + \mu_t f_x$ to be written as

$$\bar{\pi}_t + \mu_t f_x = \frac{\delta_f f_e}{1-T(a_t^{in})} + \frac{1-T(a_t^x)}{1-T(a_t^{in})}f_x$$

Hence the change in $\bar{\pi}_t + \mu_t f_x$ can be analyzed by studying the components

$$\begin{aligned}
g(a) &= \frac{adT(a)/da}{1-T(a)} > 0 \\
t(a_t^{in}, a_t^x) &= \frac{[1-T(a_t^x)]f_x}{[1-T(a_t^{in})][1+\kappa(a_t^{in})]f_p + [1-T(a_t^x)][1+\kappa(a_t^x)]f_x} \in (0,1) \\
\varepsilon_t^{in} &= \frac{da_t^{in}}{df_x}\frac{f_x}{a_t^{in}} = -\frac{t(a_t^{in}, a_t^x)}{\sigma-1} < 0 \\
\varepsilon_t^x &= \frac{da_t^x}{df_x}\frac{f_x}{a_t^x} = \frac{1-t(a_t^{in}, a_t^x)}{\sigma-1} > 0
\end{aligned}$$

Since the average profit increases following a trade liberalization, a sufficient (although not necessary) condition for higher $\bar{\pi}_t + \mu_t f_x$ is that $d(\mu_t f_x)/df_x \leq 0$ which is true if and only if

$$-g(a_t^x)\varepsilon_t^x + g(a_t^{in})\varepsilon_t^{in} + 1 \leq 0 \ ,$$

which substituting for the elasticities simplifies to

$$\left(1 - t(a_t^{in}, a_t^x)\right)g(a_t^x) + t(a_t^{in}, a_t^x)g(a_t^{in}) \geq \sigma - 1 \ .$$

The left hand side is a convex combination of $g(a_t^{in})$ and $g(a_t^x)$, hence the condition is satisfied for $min\left\{g(a_t^{in}), g(a_t^x)\right\} \geq \sigma - 1$. Assuming that $g(a)$ is an increasing function over the entire support $(0, \infty)$ yields the sufficient condition

$$g(a_t^{in}) \geq \sigma - 1 \implies \frac{d(\bar{\pi}_t + \mu_t f_x)}{df_x} < 0 \ .$$

The regularity condition on $g(a)$ applies to a large class of distributions which are commonly used to fit the c.d.f. of total factor productivity across firms.[30] And when such a property is met then the sufficient condition for $\frac{d(\bar{\pi}_t + \mu_t f_x)}{df_x} < 0$ amounts to assume that the exogenous minimum value of productivity is sufficiently large compared to the elasticity of substitution across varieties.

The effect of a decrease in the variable trade cost τ on a_t^{in} and a_t^x follows the same approach than a decrease in the fixed cost. Furthermore, the comparative statics of a change in $\bar{\pi}_t + \mu_t f_x$ holds without further restriction, since both $\bar{\pi}_t$ and μ_t increase when the variable trade cost falls, while the fixed cost is unchanged.

7.3 Welfare

Let $\bar{a}_t^d$ be the productivity of the firm which charges a price on the domestic market $\bar{p}_t^d$ and serves a domestic demand $\bar{q}_t^d$ making the average revenue per variety $\bar{p}_t^d \bar{q}_t^d = \frac{R_t}{(1+\mu_t)M_t} = \frac{\bar{r}_t}{1+\mu_t} = \bar{r}_t^d$. Substituting in the demand function (1), with $R_t = P_t Q_t$, allows the consumption based price index and the indirect utility from consumption to be written as: $P_t = [(1+\mu_t)M_t]^{-\frac{1}{\sigma-1}} \bar{p}_t^d$ and $Q_t = [(1+\mu_t)M_t]^{\frac{\sigma}{\sigma-1}} \bar{q}_t^d$. Let $\bar{w}_t^d$ be the wage paid by a firm endowed with productivity $\bar{a}_t^d$ then the productivity of the representative firm satisfies $\left(\frac{\bar{a}_t^d}{a_t^{in}}\right)^{\sigma-1} = \frac{\bar{r}_t^d}{r_{0t}} \left(\frac{\bar{w}_t^d}{w_{0t}}\right)^{\sigma-1}$. Substituting for $\bar{p}_t^d \bar{q}_t^d = \frac{2\sigma-1}{\sigma-1} \frac{(N-u_t)\bar{w}_t}{(1+\mu_t)M_t}$ and $r_{0t} = \frac{2\sigma-1}{\sigma-1} \frac{f_e}{k} w_{0t} h_{0t}$ yields

$$\bar{a}_t^d = \left(\frac{1}{(1+\mu_t)M_t}\right)^{\frac{1}{\sigma-1}} \left(\frac{N-u_t}{h_{0t} f_e/k} \frac{\bar{w}_t}{w_{0t}}\right)^{\frac{1}{\sigma-1}} \frac{\bar{w}_t^d}{w_{0t}} \ a_t^{in}$$

Substituting for average employment and for (20) yields

$$\frac{\bar{w}_t^d}{\bar{a}_t^d} = \left(\frac{1+\mu_t}{1+\mathbf{1}(\bar{a}_t)\tau^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{\bar{w}_t}{\bar{a}_t}$$

The price the representative firm charges on the domestic market is $\bar{p}_t^d = \frac{2\sigma-1}{\sigma-1} \frac{\bar{w}_t^d}{\bar{a}_t^d}$ which implies that the quantity sold in the domestic market $\bar{q}_t = \frac{\bar{r}_t/\bar{p}_t^d}{1+\mu_t}$ is proportional to the demand served by the cutoff firm $q_{0t} = a_t^{in} h_{0t} f_e/k$ as follows

$$\bar{q}_t^d = \frac{N-u_t}{(1+\mu_t)M_t} \frac{\bar{w}_t}{\bar{w}_t^d} \ \bar{a}_t^d = \left(\frac{1}{(1+\mu_t)M_t}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{N-u_t}{h_{0t} f_e/k} \frac{\bar{w}_t}{w_{0t}}\right)^{\frac{\sigma}{\sigma-1}} \quad q_{0t}$$

Substituting in the expression for aggregate indirect utility yields welfare (25).

[30]See Melitz (2003) footnote 15 for a discussion.

7.4 Discussion on the impact of trade

Unemployment and job finding probability jump at the time of implementation and then converge to the same steady state level: $u_i > u_B = u_A$ and $x_i < x_B = x_A$. During the transition both variables follow a monotonic path: $u_i > ... > u_t > u_{t+1} > ... > u_A$ and $x_i < ... < x_t < x_{t+1} < ... < x_A$.

The equation for the cutoff wage (19) shows that the revenue of the cutoff firm, which is proportional to $w_{0t}h_{0t}$, is a decreasing function of the product $\lambda(u_t)h_{0t}$. The variable $\lambda(u_t)$ is decreasing in unemployment, while $h_{0t} = (1 - x_t)(1 - \phi x_t)$ is decreasing in job finding probability, hence the interpretation of the change in the product $\lambda(u_t)h_{0t}$ is not immediate. However, substituting (10) and (12) in the definition of $\lambda(u_t)$ yields $\lambda(u_i)h_{0i} = (1 - \delta_j)\frac{(1-x_i)(1-\phi x_i)}{1-x_{i+1}}$. Since $\frac{1-x_i}{1-x_{i+1}} > 1$ then $\lambda(u_i)h_{0i} > (1 - \delta_j)(1 - \phi x_i) > (1 - \delta_j)(1 - \phi x_B) = \lambda(u_B)h_{0B} = \lambda(u_A)h_{0A}$. Therefore,

$$w_{0i}h_{0i} < w_{0B}h_{0B} = w_{0A}h_{0A} \ ,$$

and the dynamics of $w_{0t}h_{0t}$ exhibits the same increasing convergence of the job finding probability. The opposite is true for the product $\lambda(u_t)h_{0t}$. This result implies that the wage of the cutoff firm follows the same path of the job finding probability:

$$\begin{aligned} w_{0i} &< w_{0B} \quad \text{and} \quad w_{0i} < ... < w_{0t} < w_{0t+1} < ... < w_{0A} = w_{0B} \\ h_{0i} &> h_{0B} \quad \text{and} \quad h_{0i} > ... > h_{0t} > h_{0t+1} > ... > h_{0A} = h_{0B} \end{aligned}$$

The implementation of a trade liberalization induces a selection of the small firms out of the market. Hiring decisions were predetermined, and starting from a steady state allocation they were simply aimed to compensate exogenous job destruction which hits uniformly across the wage distribution. Therefore among the remaining firms, average employment per firm is higher and the employment level at the new cutoff is closer to the average employment than before:

$$h(\bar{w}_i; z_i) > h(\bar{w}_B; z_B) \quad \text{and} \quad \frac{h(\bar{w}_i; z_i)}{h_{0i}} < \frac{h(\bar{w}_B; z_B)}{h_{0B}} \ .$$

Substituting the cutoff wage (19) in the equation for the equilibrium hiring rate (16) shows that the employment ratio $h(w; z_t)/h_{0t}$ is increasing in both the wage ratio w/w_{0t} and the product $\lambda(u_t)h_{0t}$. Since $\lambda(u_i)h_{0i} > \lambda(u_B)h_{0B}$ then a lower employment ratio can be implied only by a lower wage ratio. Moreover since $w_{0i} < w_{0B}$ then the average wage is also lower in absolute terms:

$$\bar{w}_i < \bar{w}_B \quad \text{and} \quad \frac{\bar{w}_i}{w_{0i}} < \frac{\bar{w}_B}{w_{0B}} \ .$$

As shown in (20), the average productivity ratio is fixed by $\frac{h(\bar{w}_i;z_i)}{h_{0i}}\left(\frac{\bar{w}_i}{w_{0i}}\right)^\sigma < \frac{h(\bar{w}_B;z_B)}{h_{0B}}\left(\frac{\bar{w}_B}{w_{0B}}\right)^\sigma$ which determines the average profit gross of the cost for export participation $\bar{\pi}_t + \mu_t f_x$. Given that the average cost of export participation does not decrease after the trade liberalization, the average profit of a domestic firm is lower at the time of implementation,

$$\bar{\pi}_i < \bar{\pi}_B \ .$$

In the long run, unemployment, job finding probability and indeed the extremes of the wage support are equal to the initial steady state. Therefore, the equilibrium hiring rate yields the same map from wage to employment than before the trade liberalization but the higher productivity cutoff implies higher wage, by (20) which in turn yields higher average employment. Again the equilibrium condition (20) evaluated in the long run explains the higher average profit:

$$h(\bar{w}_A; z_A) > h(\bar{w}_B; z_B) \quad , \quad \bar{w}_A > \bar{w}_B \quad \text{and} \quad \bar{\pi}_A > \bar{\pi}_B \ .$$

The mass of incumbent firms at the time of implementation is given by

$$M_i = \left[1 - T\left(a_A^{in}\right)\right]\left(E_B + \frac{1-\delta_f}{1-T\left(a_B^{in}\right)} M_B\right) \qquad \text{where} \qquad E_B = \frac{\delta_f M_B}{1-T\left(a_B^{in}\right)}$$

because it is the mass of entrants which would compensate for exogenous firm exit, starting from the initial steady state. Hence $M_i = \frac{1-T\left(a_A^{in}\right)}{1-T\left(a_B^{in}\right)} M_B < M_B$ and total profit is $\bar{\pi}_i M_i < \bar{\pi}_B M_B$. Therefore the total number of potential entrants which can be financed at the time of implementation is less than before $E_i < E_B$. Substituting for $[1 - T\left(a_B^{in}\right)] = (M_B/E_B)\delta_f$ and $[1 - T\left(a_A^{in}\right)] = (M_A/E_A)\delta_f$ in (6) yields

$$\frac{M_i}{M_B} = \frac{M_A}{M_B}\left[\frac{E_i}{E_A}\delta_f + \frac{E_B}{E_A}(1-\delta_f)\right] \quad \text{and} \quad \frac{M_{i+1}}{M_i} = \frac{M_A}{E_A}\frac{E_i}{M_i}\delta_f + (1-\delta_f)$$

The first equation is a sufficient condition for $E_i < E_B \implies M_i < M_A$. The second equation is informative about the evolution of the mass of firms when it is interpreted jointly with the condition for financing entry, which implies

$$\frac{E_i}{M_i} \quad = \quad \frac{\bar{\pi}_i}{f_e} < \frac{E_B}{M_B} = \frac{\bar{\pi}_B}{f_e} < \frac{E_A}{M_A} = \frac{\bar{\pi}_A}{f_e} \ .$$

At the time of implementation the mass of firms is not in a stable point. And substituting for $E_i < \delta_f M_i / [1 - T\left(a_A^{in}\right)]$ in (6) shows that the next period mass of incumbent firms is even lower $M_{i+1} < M_i$. Given the same selection (the productivity cutoff is unchanged) the mass of incumbent firms is even lower than at the time of implementation. This boosts the average profit and indeed the value of entry from the following period on, above the level at which the free entry condition would be satisfied. This explains why the mass of firms increases converging to M_A from below, as the share of entrants is above the one in steady state $E_t/M_t \geq E_A/M_A$ for every period after the implementation. Hence the average profit and the expected value of entry are larger than in the long run steady state during the entire transition.

Change in welfare. Notice that $\frac{h(\bar{w}_i;z_i)}{h_{0i}} < \frac{h(\bar{w}_B;z_B)}{h_{0B}}$ and $\frac{\bar{w}_i}{w_{0i}} < \frac{\bar{w}_B}{w_{0B}}$ have several implications. First, through the wage equation, they imply $\frac{\bar{r}_i}{r_{0i}} < \frac{\bar{r}_B}{r_{0B}}$. Hence, since $w_{0i} h(w_{0i}, z_i) < w_{0B} h(w_{0B}, z_B)$ and $M_i < M_B$ then average and aggregate revenue fall

$$\bar{r}_i < \bar{r}_B \quad \text{and} \quad \bar{R}_i < \bar{R}_B \ .$$

This also implies that the average revenue of the representative domestic firm is lower $\bar{r}_i^d < \bar{r}_B^d$, as $\mu_i > \mu_B$. To the end of understanding the change in welfare at the implementation, I use the definition of aggregate revenue $P_t Q_t = R_t$ and the definition of consumption based price index to obtain the price ratio:

$$\frac{P_B}{P_i} = \left(\frac{M_i}{M_B}\right)^{\frac{1}{\sigma-1}} \left(\frac{1+\mathbf{1}(\bar{a}_i)\tau^{1-\sigma}}{1+\mathbf{1}(\bar{a}_B)\tau^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{\bar{a}_i}{\bar{a}_B}\frac{\bar{w}_B}{\bar{w}_i}$$

Then, the condition (20) allow to substitute for

$$\left(\frac{1+\mathbf{1}(\bar{a}_i)\tau^{1-\sigma}}{1+\mathbf{1}(\bar{a}_B)\tau^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{\bar{a}_i}{\bar{a}_B} = \left(\frac{h(\bar{w}_i;z_i)/h_{0i}}{h(\bar{w}_B;z_B)/h_{0B}}\right)^{\frac{1}{\sigma-1}} \left(\frac{\bar{w}_i/w_{0i}}{\bar{w}_B/w_{0B}}\right)^{\frac{\sigma}{\sigma-1}} \frac{a_i^{in}}{a_B^{in}} < \frac{a_i^{in}}{a_B^{in}}$$

which implies:

$$\frac{P_B}{P_i} < \left(\frac{M_i}{M_B}\right)^{\frac{1}{\sigma-1}} \frac{\bar{w}_B}{\bar{w}_i}\frac{a_i^{in}}{a_B^{in}} = \left(\frac{M_i}{M_B}\right)^{\frac{1}{\sigma-1}} \frac{p_{0B}}{p_{0i}} < \frac{p_{0B}}{p_{0i}} \; .$$

Therefore, the change in welfare can be investigated by looking at $\frac{Q_i}{Q_B} = \frac{R_i}{R_B}\frac{P_B}{P_i}$, which yields

$$\frac{Q_i}{Q_B} < \frac{R_i}{R_B}\frac{p_{0B}}{p_{0i}} = \left(\frac{R_i}{R_B}\right)^{1-\frac{1}{\sigma-1}} \left(\frac{r_{0i}}{r_{0B}}\right)^{\frac{1}{\sigma-1}}$$

where the equality is obtained by substituting for $\frac{p_{0B}}{p_{0i}} = \left(\frac{R_B}{R_i}\right)^{\frac{1}{\sigma-1}} \left(\frac{r_{0i}}{r_{0B}}\right)^{\frac{1}{\sigma-1}}$ as implied by the demand equation (1). The wage equation (5) can then be used to rewrite the ratio in revenues

$$\frac{Q_i}{Q_B} < \left(\frac{R_i}{R_B}\right)^{\frac{\sigma-2}{\sigma-1}} \left(\frac{w_{0i}h_{0i}}{w_{0B}h_{0B}}\right)^{\frac{1}{\sigma-1}} .$$

Since $w_{0i}h_{0i} < w_{0B}h_{0B}$ and $R_i < R_B$ a sufficient condition for $\frac{Q_i}{Q_B} < 1$ is $\sigma \geq 2$, (although it is not a necessary condition).

7.5 Parameter restrictions

Two conditions must be checked. First, the hiring rate should be an increasing function of the wage for all $w \geq w_{0t}$. This boils down to check whether the denominator of 16 is a decreasing function of the wage ratio w/w_{0t}. This is true if and only if $\sigma(\alpha(z_t)+\beta(z_t)-1)\,(w/w_{0t})^{\sigma-1} \geq \beta(z_t)$. Since $w \geq w_{0t}$ and $(\alpha(z_t)+\beta(z_t)-1)f_p/f_e$ from the zero profit condition, then:

$$\beta(z_t) \leq \sigma\frac{f_p}{f_e} \implies \frac{dh(w;z_t)}{dw} \geq 0 \; , \; \forall w \geq w_{0t}$$

Rewriting $\beta(z_t) = \frac{\sigma}{\sigma-1}\frac{w_{0t}h_{0t}}{k}$, and using the equation (19) for the minimum wage w_{0t} where $h_{0t} = (1-x_t)(1-\phi x_t)$ yields the equivalent sufficient condition $\lambda(u_t)h_{0t} \geq 1-(\sigma-1)\frac{f_p}{f_e}$. Since $x_t < 1$ and $\phi \leq 1$ by construction such that $h_{0t} > 0$ and $\lambda(u_t) > 0$ then

$$f_p \geq \frac{f_e}{\sigma-1}$$

is a sufficient (although not necessary) condition for the existence of a wage dispersion equilibrium.

A necessary and sufficient condition for selection of firms in the export market is

$$f_x > \tau^{-(\sigma-1)} f_p$$

such that $a_t^x > a_t^{in}$. The two restrictions $f_e > 0$ and $k > 0$ guarantee a positive but finite number of vacancies, $\delta_j > 0$ yields a positive unemployment rate, $\delta_f > 0$ yields a stable mass of firms and $\delta = \delta_f + \delta_j - \delta_f \delta_j < \frac{1}{2}$ guarantees a steady state unemployment rate which is less than 1.

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